

TAP 322- 3: Grating calculations

These questions give you practice in using the grating formula $n \lambda = d \sin \theta_n$.

A grating is labelled '500 lines per mm'.

1. Calculate the spacing of the slits in the grating.
2. Monochromatic light is aimed straight at the grating and is found to give a first-order maximum at 15° . Calculate the wavelength of the light source.
3. Calculate the position of the first-order maximum when red light of wavelength 730 nm is shone directly at the grating.
4. The longest visible wavelength is that of red light with $\lambda = 750$ nm. The shortest visible wavelength is violet where $\lambda = 400$ nm. Use this information to calculate the width of the angle into which the first-order spectrum is spread out when white light is shone onto the grating.

A grating is illuminated with a parallel beam of light of wavelength 550 nm. The first-order maximum is in a direction making an angle of 20° with the straight-through direction.

5. Calculate the spacing of the grating slits.
6. What would be the angle of the first-order maximum if a grating of slit spacing of

2.5×10^{-6} m were used with the same light source?

7. Calculate the wavelength of light that would give a second-order maximum at $\theta = 32^\circ$ with a grating of slit spacing 2.5×10^{-6} m.

Hints

1. What must the gap be between the centre of each line in order to fit 500 lines into 1 mm? Remember to express your answer in metres.
2. This is about the first-order minimum so use the formula $n\lambda = d \sin \theta$ with $n = 1$.
3. Rearrange the formula $n\lambda = d \sin \theta$ to make $\sin \theta$ the subject. Remember to take the arcsin (or \sin^{-1}) to give an answer in degrees.
4. Use the same method as question 3 to obtain the position of first-order maxima for red and violet light. The dispersion is simply the angle of maximum of red light minus the angle of maximum of violet light.

Practical advice

A set of simple practice questions, to build confidence.

Social and human context

These questions introduce the idea of interference gratings used to obtain spectra. This has been extremely important in the development of modern astronomy.

Answers and worked solutions

1. Number of slits per mm = 500. Therefore, slit spacing = $1 / 500 = 0.002 \text{ mm} = 2 \times 10^{-6} \text{ m}$.

2.

$$n \lambda = d \sin \theta_n.$$

Therefore

$$\lambda = 2 \times 10^{-6} \times \sin 15^\circ = 5.2 \times 10^{-7} \text{ m or } 520 \text{ nm}.$$

3.

$$n \lambda = d \sin \theta_n.$$

Therefore

$$\sin \theta = \lambda / d = 0.358$$

from which $\theta = 21^\circ$.

4.

$$n \lambda = d \sin \theta.$$

For red light:

$$\sin \theta_r = \lambda / d = 7.5 \times 10^{-7} \text{ m} / 2 \times 10^{-6} \text{ m} = 0.375$$

angle $\theta_r = 22^\circ$.

For violet light:

$$\sin \theta_v = \lambda / d = 4.0 \times 10^{-7} \text{ m} / 2 \times 10^{-6} \text{ m} = 0.20$$

angle $\theta_v = 12^\circ$. The difference in angle is 10° .

5.

$$d = \lambda / \sin \theta.$$

$$d = 5.5 \times 10^{-7} \text{ m} / 0.342 = 1.6 \times 10^{-6} \text{ m}.$$

6.

$$\sin \theta = \lambda / d.$$

$$\sin \theta = 5.5 \times 10^{-7} \text{ m} / 2.5 \times 10^{-6} \text{ m} = 0.22.$$

Thus

$$\theta = 12.7^\circ.$$

7.

$$n \lambda = d \sin \theta.$$

Thus

$$\begin{aligned}\lambda &= (2.5 \times 10^{-6} \text{ m} \times \sin 32^\circ) / 2 \\ &= (2.5 \times 10^{-6} \text{ m} \times 0.530) / 2 \\ &= 6.6 \times 10^{-7} \text{ m} = 660 \text{ nm}.\end{aligned}$$

External reference

This activity is taken from Advancing Physics chapter 6, 200S