TAP 305- 5: Energy and pendulums

Questions

A body of mass 100 g undergoes simple harmonic motion with amplitude of 20 mm. The maximum force which acts upon it is 0.05 N. Calculate:

1. Its maximum acceleration.

2. Its period of oscillation.

A baby in a 'baby bouncer' is a real-life example of a mass-on-spring oscillator. The baby sits in a sling suspended from a stout rubber cord, and can bounce himself up and down if his feet are just in contact with the ground. Suppose a baby of mass 5.0 kg is suspended from a cord with spring constant 500 N m^{-1}. Assume g = 10 N kg^{-1}.

3. Calculate the initial (equilibrium) extension of the cord.

4. What is the value of ω (= $2\pi / T$)?

5. The baby is pulled down a further distance, 0.10 m, and released. How long after his release does he pass through his equilibrium position?

6. What is the maximum speed of the baby?

A simple pendulum has a period of 4.2 s. When it is shortened by 1.0 m the period is only 3.7 s.

7. Without assuming a value for g, calculate the original length of the pendulum.

8. Calculate the acceleration due to gravity g suggested by the data.

Practical advice

These are rather harder questions on simple harmonic motion, involving numerical, and in the case of the last two, algebraic competence.

Answers and worked solutions

1.

$$a = \frac{F}{m} = \frac{0.05\ \text{N}}{10\ \text{kg}} = 0.5\ \text{m s}^{-2}$$

for 10 kg read 0.1 kg

2.

$$a = \omega^2 s$$

and

$$\omega = \frac{2\pi}{T}$$

so

$$T = 2\pi\sqrt{\frac{s}{a}} = 2\pi\sqrt{\frac{20 \times 10^{-3}\ \text{m}}{0.5\ \text{m s}^{-2}}} = 1.3\ \text{s}.$$

3.

$$x = \frac{F}{x} = \frac{mg}{x} = \frac{5.0\ \text{kg} \times 10\ \text{N kg}^{-1}}{500\ \text{N m}^{-1}} = 0.1\text{m}$$

4.

$$\omega = \frac{2\pi}{T}$$

and

$$T = 2\pi\sqrt{\frac{m}{k}}$$

so

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500\ \text{N m}^{-1}}{5.0\ \text{kg}}} = 10\ \text{rad s}^{-1}$$

5.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\ \text{rad}\,\text{s}^{-1}}$$

and

$$\frac{T}{4} = \frac{2\pi}{4 \times 10\ \text{rad s}^{-1}} = 0.16\ \text{s}$$

6.

$$v_{\text{max}} = 2\pi fA = \omega A = 10\ \text{rad s}^{-1} \times 0.10\ \text{m} = 1.0\ \text{m}\,\text{s}^{-1}.$$

7. With two unknowns, this requires the use of simultaneous equations. Initially

$$T = 2\pi\sqrt{\frac{l}{g}} = 4.2\ \text{s}$$

When the length is shorter

$$2\pi\sqrt{\frac{l - 1.0\ \text{m}}{g}} = 3.7\ \text{s}.$$

Dividing one equation by the other

$$\frac{\sqrt{\frac{l}{g}}}{\sqrt{\frac{l - 1.0\ \text{m}}{g}}} = \frac{4.2\ \text{s}}{3.7\ \text{s}}.$$

This gives

$$\frac{l}{l - 1.0\ \text{m}} = \left(\frac{4.2}{3.7}\right)^2 = 1.29$$

$$l = 1.29(l - 1.0\ \text{m}) = 1.29l - 1.29\ \text{m}$$

and

$$0.29l = 1.29\ \text{m}$$

so

$$l = \frac{1.29\ \text{m}}{0.29} = 4.5\ \text{m}.$$

8. Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 (4.5\ \text{m})}{(4.2\ \text{s})^2} = 10\ \text{m}\,\text{s}^{-2}.$$

External reference

This activity is taken from Advancing Physics chapter 10, 170S