Episode 305: Energy in SHM

Qualitatively, students will appreciate that there is a continuous interchange between potential and kinetic energy during SHM. Here, they can also learn about the mathematical basis for calculating energy.

Summary

Demonstration: An experimental displacement-time graph. (10 minutes)

Discussion: Maximum values of quantities in SHM. (15 minutes)

Student questions: Practice with the equations. (30 minutes)

Discussion: Energy changes in SHM. (20 minutes)

Student questions: Energy of a pendulum. (20 minutes)

Demonstration:

An experimental displacement-time graph

Use a ‘water pendulum’ to draw a large displacement-time graph for a pendulum. You could ask a group of students to prepare this in advance and demonstrate it to the class.

TAP 305-1: The water pendulum

Discussion:

Maximum values of quantities in SHM

Refer back to the sin and cos equations for SHM. Show that the maximum values of displacement, velocity and acceleration are given by (the term in front of sin or cos):

- maximum displacement = A
- maximum velocity = $A\omega$
- maximum acceleration = $A\omega^2$
- Compare these relationships with the equations for circular motion:
- displacement = r
- velocity = $r\omega$
- acceleration = $r\omega^2$

If you have adopted the ‘auxiliary circle’ approach earlier, the parallels should be clear.

Student questions:

Practice with the equations

It will help to provide some more practice in using the equations and analyzing motion.

TAP 305-2: Oscillators

Discussion:

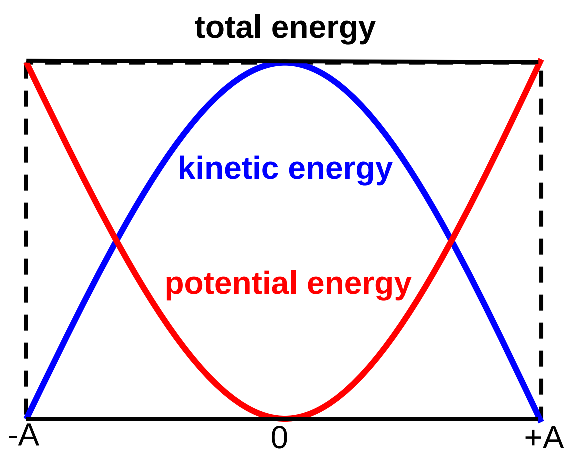
Energy changes in SHM

Think about the energy changes in a mechanical oscillator. Recap that, as it passes through its equilibrium position, its speed and hence its kinetic energy are a maximum. At the maximum displacement, the speed and hence the kinetic energy are both zero. The potential energy will be a maximum when the speed is zero and vice versa. Assuming that there is no friction or air drag the total energy input *E* of the oscillator must remain constant

For the mass and spring system, the work done stretching a spring by an amount *x* is the area under the force extension graph = 1/2 kx^2. The PE-extension graph is a parabola.

The kinetic energy will be zero at +*A* and a maximum when *x* = 0, so its graph is an inverted version of the strain energy graph. At any position kinetic + elastic strain energy is a constant *E*, where $E = KE_{max} = PE_{max}$.

$PE_{max} \propto A^2$, so the total energy *E* of SHM is proportional to (amplitude) ^{2}.



TAP 305-3: Elastic energy

Draw a graph from *x* = {–*A* to +*A*}, to show kinetic energy, strain energy and total energy. You can also draw graphs of KE and PE against time.

TAP 305-4: Energy flow in an oscillator

Student questions:

Energy of a pendulum

Some useful questions on energy of a pendulum.

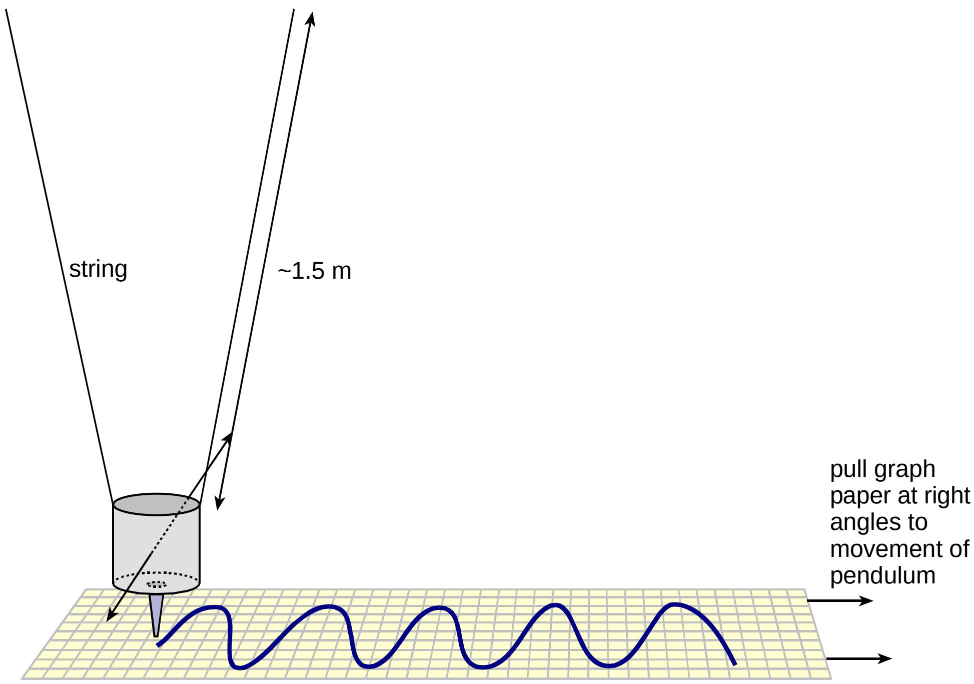
TAP 305-5: Energy and pendulums

TAP 305- 1: The water pendulum

As inky water runs out of a hole in the bottom of a can, it can create a trace on paper slowly moved underneath. Although this is a ‘rough and ready’ way of recording the oscillation, it can give you useful information.

You will need

- ✓ water pendulum
- ✓ four sheets of graph paper taped together, or light-coloured sugar paper cut to size
- ✓ two retort stands, bosses and clamps
- ✓ two G clamps, approximately 5 cm jaw
- ✓ ink
- ✓ hand-held stopwatch



What to do

You want the pendulum to swing with an amplitude of half the width of the graph paper. As it is swinging, you are going to drag the paper underneath the dribbling water to create a time trace. To do this you will need to pull the paper with constant velocity, so do a few ‘dry runs’ before putting water in the can.

Make sure the end of the tube is as near as possible to the paper at the bottom of the swing.

1. Put your finger over the hole and put about 2 cm of water in the can. Now add 10 drops of ink and allow it to mix.

2. Set the pendulum swinging and slowly pull the graph paper under the water. This will give you a time trace. Let it dry and add axes to the line or make a good copy on another sheet of graph paper.

Now you have done the experiment, think how to answer these questions:

- How can you mark time values on the graph?
- Does the periodic time of the oscillation change as the water runs out?
- What is the maximum velocity of the pendulum?

You have seen

1. The pendulum reaches its maximum velocity when it travels through its mean position.
2. The time trace is a displacement–time graph so velocity at any instant can be found by taking the gradient at that instant.

Practical Advice

If a little care is taken, this activity will yield a reasonable time trace. The aim of this activity is getting your students to think more carefully about the nature of oscillators. Producing a time trace should encourage students to observe more carefully. The graph obtained should be clear enough for them to consider how the gradient varies over the cycle.

The ‘water pendulum’ needs to be made up. At its simplest it is just a can with a 1 mm hole in the centre. A more reliable effect is produced when a glass tube is inserted into a bung and then the bung in the bottom of the can.

The supporting strings should be about 1.5 m long.

Safety

If the paper is on the bench, the string supports are roughly 2.5 m from the floor. This requires two persons; one to hold the ladder or steps and the other to fix the string supports. If the paper is on the floor, the retort stands can be on two tables; ensure that they cannot move.

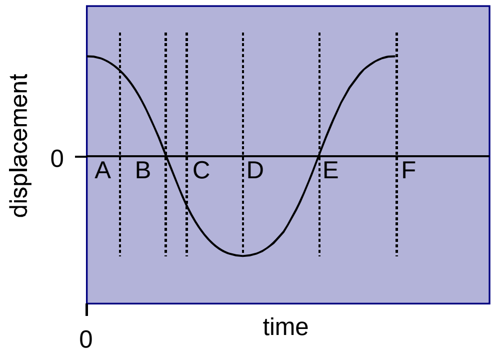
External reference

This activity is taken from Advancing Physics chapter 10, 200P

TAP 305- 2: Oscillators

Questions

Here is the displacement–time graph of an oscillator.



1. Consider the speed of the oscillator at the four times labelled *A*, *B*, *C*, and *D*. Arrange the times *A*, *B*, *C*, *D* in order of decreasing speed.

2. How does the velocity at time *B* compare with that at time *E*?

3. How does the velocity at time *D* compare with that at time *F*?

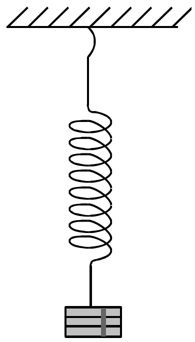
4. At which of the times 0 to *F* is the acceleration at its largest value?

5. At which of the times 0 to *F* is the displacement equal in size to the amplitude of the motion?

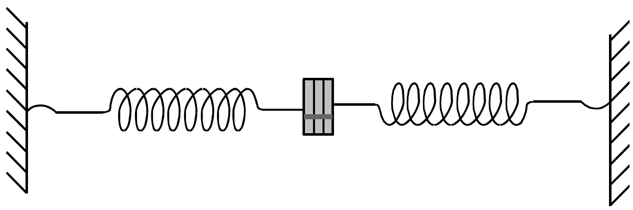
6. Consider the time intervals 0–*B*, 0–*D*, 0–*F*, *B*–*E*, *D*–*F*. If the periodic time of the oscillator is *T*, write down each interval in terms of *T*. (0–*F* = 3 *T* is the sort of answer expected, though this particular answer would be wrong.)

Here are three things which would oscillate in a laboratory on Earth.

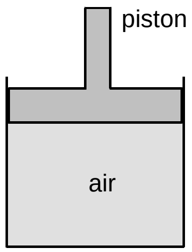
(a)



(b)



(c)



7. Which, if any, would oscillate in a spacecraft going at steady speed a long way from the Earth and from any planet or star?

Explain your answer.

Sketch a large displacement–time graph for two periods of a simple harmonic motion.

8. Mark with M any instant where the speed is a maximum

9. Mark with Z any instant when the speed is zero.

10. Mark places where the acceleration is high H and where it is low L.

Answer each of the following, giving reasons:

11. Does a tuning fork, used by musicians, vibrate with simple harmonic motion?

12. Is the bouncing of a ball a simple harmonic motion?

13. If a pendulum were taken to the top of a mountain, would it gain or lose time?

Practical advice

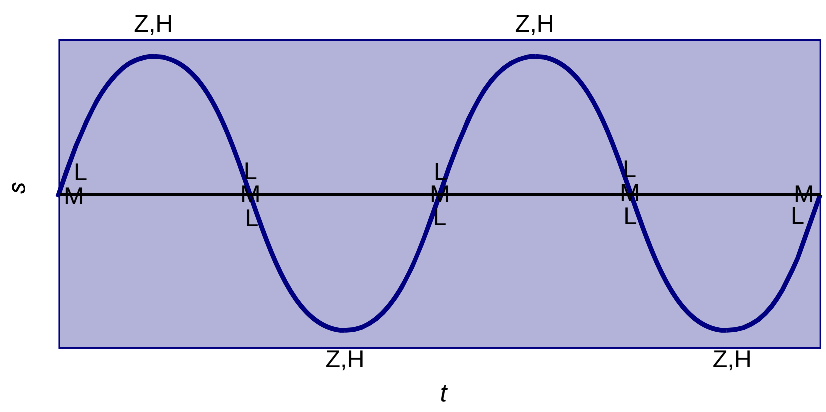
These questions are concerned with basic ideas about simple harmonic motion. They may therefore prove useful for class discussion. Others provide practice identifying and using relevant equations.

Answers

1. B C A D
2. Same magnitude, opposite direction.
3. Both zero.4. 0, D and F
5. 0, D and F
6. 0–B = T 4, 0–D = T/2, 0–F = T, B–E =T/2, D–F = T/2
7. (b) will oscillate. Only the horizontal mass-spring system does not depend on gravity.

8., 9. and 10.

Note again that *a* is proportional to displacement, but in the opposite direction.



11. Yes. For small displacements, this is an instance of Hooke's law, with the force proportional to the displacement.
12. No. The force (and acceleration) is constant when removed from the surface, being the weight of the ball.
13. Loses time.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

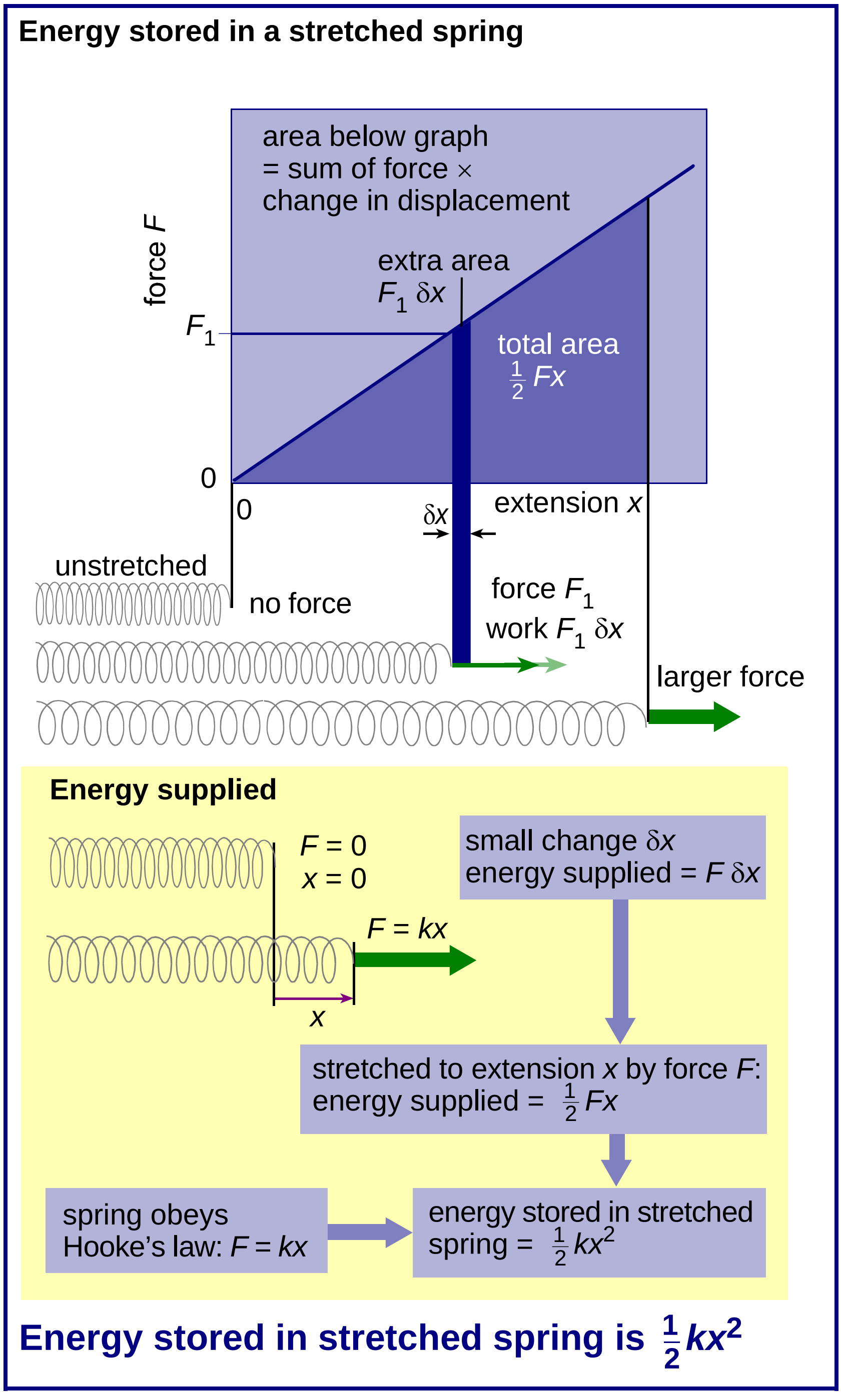
and *g* decreases with height (distance from Earth's centre), so *T* will increase.

External reference

This activity is taken from Advancing Physics chapter 10, 160S

TAP 305- 3: Elastic energy

The relationship between force to extend a spring, and extension, determines the energy stored.



Practical advice

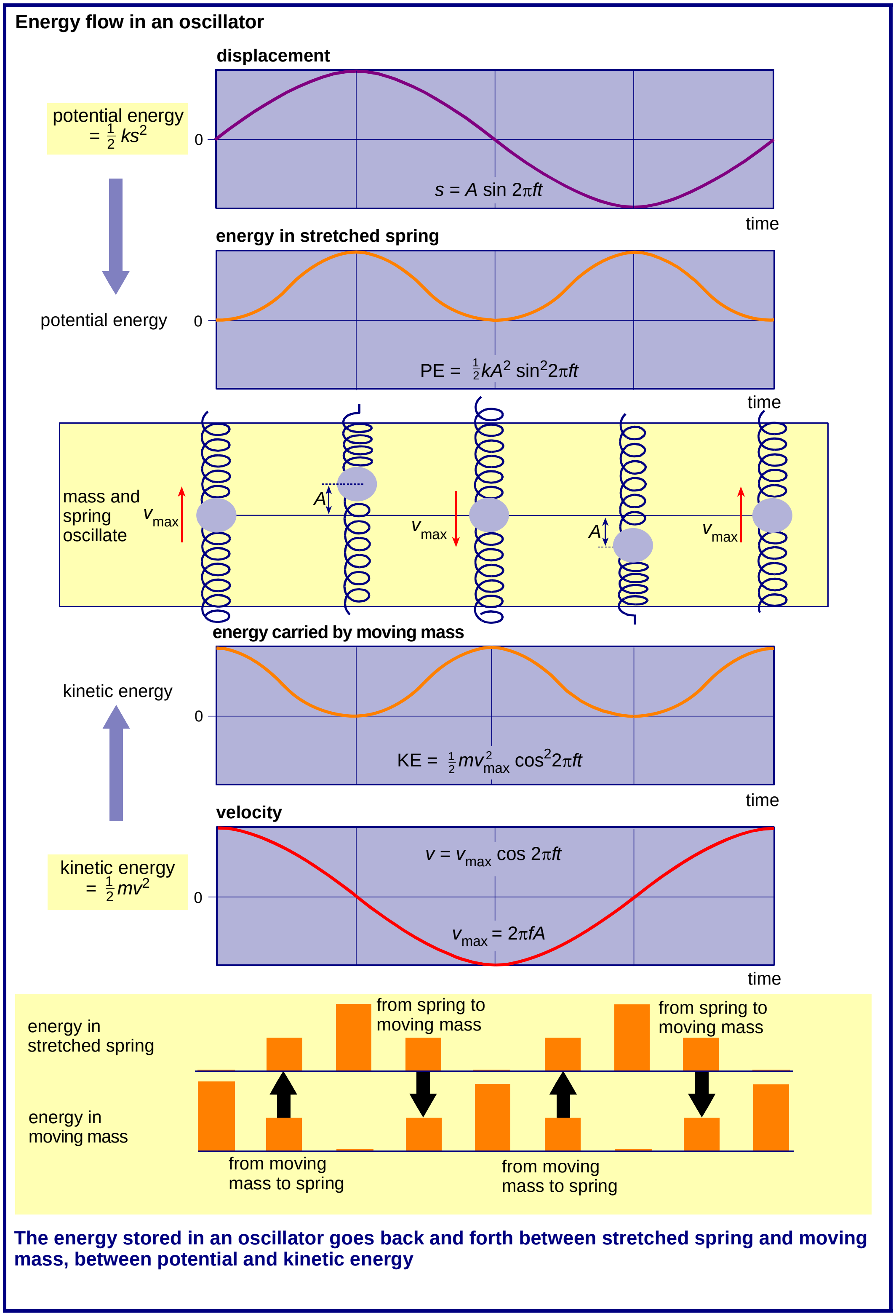
This diagram is reproduced here so that you can talk through it, or adapt it to your own purposes.

External reference

This activity is taken from Advancing Physics chapter 10, 120O

TAP 305- 4: Energy flow in an oscillator

The energy sloshes back and forth between being stored in a spring, and carried by the mass.



Practical advice

This diagram is reproduced here so that you can talk through it, or adapt it to your own purposes.

External reference

This activity is taken from Advancing Physics chapter 10, 130O

TAP 305- 5: Energy and pendulums

Questions

A body of mass 100 g undergoes simple harmonic motion with amplitude of 20 mm. The maximum force which acts upon it is 0.05 N. Calculate:

1. Its maximum acceleration.

2. Its period of oscillation.

A baby in a ‘baby bouncer’ is a real-life example of a mass-on-spring oscillator. The baby sits in a sling suspended from a stout rubber cord, and can bounce himself up and down if his feet are just in contact with the ground. Suppose a baby of mass 5.0 kg is suspended from a cord with spring constant 500 N m^{-1}. Assume g = 10 N kg^{-1}.

3. Calculate the initial (equilibrium) extension of the cord.

4. What is the value of ω (= $2\pi / T$)?

5. The baby is pulled down a further distance, 0.10 m, and released. How long after his release does he pass through his equilibrium position?

6. What is the maximum speed of the baby?

A simple pendulum has a period of 4.2 s. When it is shortened by 1.0 m the period is only 3.7 s.

7. Without assuming a value for g, calculate the original length of the pendulum.

8. Calculate the acceleration due to gravity *g* suggested by the data.

Practical advice

These are rather harder questions on simple harmonic motion, involving numerical, and in the case of the last two, algebraic competence.

Answers and worked solutions

1.

$$a = \frac{F}{m} = \frac{0.05\ \text{N}}{10\ \text{kg}} = 0.5\ \text{m s}^{-2}$$

for 10 kg read 0.1 kg

2. $a = \omega^2 s$

and

$$\omega = \frac{2\pi}{T}$$

so

$$T = 2\pi\sqrt{\frac{s}{a}} = 2\pi\sqrt{\frac{20 \times 10^{-3}\ \text{m}}{0.5\ \text{m s}^{-2}}} = 1.3\ \text{s}.$$

3.

$$x = \frac{F}{x} = \frac{mg}{x} = \frac{5.0\ \text{kg} \times 10\ \text{N kg}^{-1}}{500\ \text{N m}^{-1}} = 0.1\,\text{m}$$

4.

$$\omega = \frac{2\pi}{T}$$

and

$$T = 2\pi\sqrt{\frac{m}{k}}$$

so

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500\ \text{N m}^{-1}}{5.0\ \text{kg}}} = 10\ \text{rad s}^{-1}$$

5.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\ \text{rad s}^{-1}}$$

and

$$\frac{T}{4} = \frac{2\pi}{4 \times 10\ \text{rad s}^{-1}} = 0.16\ \text{s}$$

6.

$$v_{max} = 2\pi f A = \omega A = 10\ \text{rad s}^{-1} \times 0.10\ \text{m} = 1.0\ \text{m s}^{-1}.$$

7. With two unknowns, this requires the use of simultaneous equations. Initially

$$T = 2\pi\sqrt{\frac{l}{g}} = 4.2\ \text{s}$$

When the length is shorter

$$2\pi\sqrt{\frac{l - 1.0\ \text{m}}{g}} = 3.7\ \text{s}.$$

Dividing one equation by the other

$$\frac{\sqrt{\frac{l}{g}}}{\sqrt{\frac{l - 1.0\ \text{m}}{g}}} = \frac{4.2\ \text{s}}{3.7\ \text{s}}.$$

This gives

$$\frac{l}{l - 1.0\ \text{m}} = \left(\frac{4.2}{3.7}\right)^2 = 1.29$$

$$l = 1.29(l - 1.0\ \text{m}) = 1.29l - 1.29\ \text{m}$$

and

$$0.29l = 1.29\ \text{m}$$

so

$$l = \frac{1.29\ \text{m}}{0.29} = 4.5\ \text{m}.$$

8. Since

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 (4.5\ \text{m})}{(4.2\ \text{s})^2} = 10\ \text{m}\,\text{s}^{-2}.$$

External reference

This activity is taken from Advancing Physics chapter 10, 170S