

## Episode 304: Simple pendulum

This episode reinforces many of the fundamental ideas about SHM.

Note a complication: a simple pendulum shows SHM only for small amplitude oscillations.

### Summary

**Student experiment: Measuring the restoring force. (20 minutes)**

**Student experiment: Testing the relationship  $T = 2\pi \sqrt{l/g}$ . (30 minutes)**

**Student activity: Using an applet of a pendulum. (30 minutes)**

**Discussion: Gravitational and inertial mass. (10 minutes)**

**Student questions: Calculations involving pendulums. (30 minutes)**

### Student experiment:

#### Measuring the restoring force

Measure the restoring force for a simple pendulum.

TAP 304-1: The simple pendulum

### Student experiment:

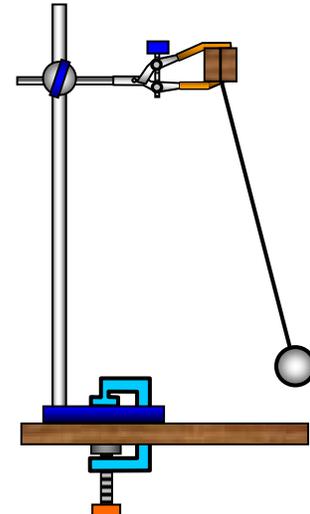
#### Testing the relationship $T = 2\pi \sqrt{l/g}$

Test the relationship  $T = 2\pi \sqrt{l/g}$  for a simple pendulum. Students could decide for themselves which measurements to make, which quantities to vary, and how to process and interpret the results. Encourage them to look for deviations from linear behaviour, arising from large-amplitude oscillations.

### Student activity:

#### Using an applet of a pendulum

Investigate a virtual pendulum; this allows you to vary  $g$ . You can also force the pendulum, which is useful later when studying resonance.



(resourcefulphysics.org)

<http://monet.physik.unibas.ch/~elmer/pendulum/index.html>

NB the analysis of the data uses log-log plots, so this may not be suitable for all students

TAP 304-2: Virtual pendulum

## **Discussion:**

### **Gravitational and inertial mass**

The fact that the period of a simple pendulum is independent of the mass of the bob is an example of the Principle of Equivalence – something still not understood today and being tested by very sophisticated experiments involving astronomical measurements on the one hand and how single atoms fall due to gravity on the other.

The basic puzzle is why the  $m$  in  $F = ma$  (where  $m$  is the inertial mass which determines how an object responds to any unbalanced force) has exactly the same magnitude as the  $m$  in  $mg$  (where the  $m$  is the gravitational mass, the source of the gravitational force).

In deriving the equation for the period of a simple pendulum, we have used both, and used the fact that numerically they cancel out.

## **Student questions:**

### **Calculations involving pendulums**

These questions reinforce basic ideas about SHM.

TAP 304-3: Pendulum

## TAP 304- 1: The simple pendulum

### Requirements

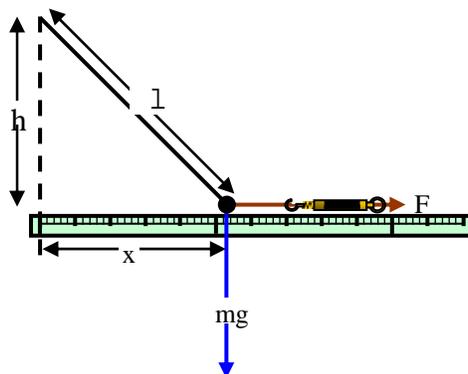
- ✓ large mass (at least 2 kg)
- ✓ very strong cord
- ✓ high, secure attachment for top of cord (tall retort stand is not suitable)
- ✓ spring balance (0–50 or 0–100 N)
- ✓ metre rule with provision for horizontal clamping

### Experimental exploration

With due regard to safety, and as far as laboratory conditions allow, hang a large known mass  $m$  (several kg if possible) from a secure attachment on 2 or 3 metres of strong cord. Clamp a metre rule horizontal and level with the mid-point of the mass when it is hanging vertically.

Attach a spring balance to the mass so that the force needed to pull it sideways can be measured (i.e. the force needed to balance the horizontal component of the tension). The diagram below shows the arrangement.

	<p><b>Safety</b></p> <p>In equilibrium the mass should be as close to the floor as possible and the cord and support should be able to withstand many times the weight of the mass. A soft landing platform must be under the mass at all times, and feet kept well clear.</p> <p><b>Wear safety spectacles</b> to protect the eyes from the whipping end of a broken cord.</p> <p><b>Two persons are needed</b> to fix a high suspension point: one to hold the ladder or steps and one to do the work.</p>
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Record the length  $l$  of the string. Take a sequence of readings ( $F$ ,  $x$ ) of the sideways force  $F$  needed to pull the mass a horizontal distance  $x$  from equilibrium. Work up to as large a value of  $x$

as you judge consistent with safety (remember, the tension in the cord will become much bigger than the weight of the mass).

Plot a graph of  $F$  against  $x$  and comment on its shape. For the linear region measure the 'stiffness'  $k$  (graph gradient) in  $\text{N m}^{-1}$ .

Use your judgement to decide the value of  $x$  above which you could consider the system non-linear. What value of angle  $\theta$  does this correspond to?

Remember,  $\sin \theta = x/l$

## Theory

Either from the triangle of forces, or by resolution horizontally and vertically, show that

$$F = mg \tan \theta$$

From  $\tan \theta = x/h$ , but provided  $x$  is not too big (small  $\theta$ ),  $h$  (which is of course variable) is approximately the same as  $l$  (which is constant), so that  $\tan \theta = x/l$

Hence

$$F = mgx/l$$

Another way of arriving at this is to use the small angle approximation that  $\tan \theta \approx \sin \theta$  for small  $\theta$ .

You can now see how good the approximation of linearity is – it is as good as you want it to be.

For  $\theta = 10^\circ$  the difference between  $\tan \theta$  and  $\sin \theta$  is 1.5% of  $\sin \theta$

For  $\theta = 20^\circ$  the difference becomes 6.4%

For  $\theta = 30^\circ$  the difference becomes 15%

You can see that the theoretical 'stiffness'  $k$  is  $mg/l$  (for the linear region).

Calculate this value and compare it with the value from the graph.

An interesting property of a pendulum is that its period is also independent of its mass. Look at the theoretical expression for  $k$  and see if you can see why this should be.

## Practical advice

Students measure the restoring force on the bob of a simple pendulum when it is displaced by different amounts. They then use trigonometry and the small angle approximation to derive an expression for the period of small-amplitude oscillations and consider what is meant here by 'small'.

	<p><b>Safety</b></p> <p>In equilibrium the mass should be as close to the floor as possible and the cord and support should be able to withstand many times the weight of the mass. A soft landing platform must be under the mass at all times, and feet kept well clear.</p> <p><b>Wear safety spectacles</b> to protect the eyes from the whipping end of a broken cord.</p> <p><b>Two persons are needed</b> to fix a high suspension point: one to hold the ladder or steps and one to do the work.</p>
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## External reference

This activity is taken from Salters Horners Advanced Physics, section BLD, additional sheet 4

## TAP 304- 2: Virtual pendulum

Use a computer simulation of a pendulum to explore factors affecting its oscillation.

Go to a suitable website with a simple pendulum simulation. For example:

<http://monet.physik.unibas.ch/~elmer/pendulum/index.html>

<http://www.walter-fendt.de/ph11e/pendulum.htm>

Use the applet to demonstrate how the period,  $T$ , of a pendulum is affected by its length,  $l$ , and/or the gravitational field strength,  $g$ .

### Varying the length

- Set  $g = 9.8 \text{ N kg}^{-1}$
- Select a value of  $l$ .
- Find  $T$  by timing a known number of oscillations.
- Change  $l$ , keeping  $g$  constant, and find the new  $T$ .
- Continue until you have at least five sets of results.
- Enter your results into a spreadsheet.
- Plot a graph of  $\log T$  against  $\log l$ .
- Find the gradient of the graph.
- Hence obtain a relationship that shows how  $T$  varies with  $l$ .

Your relationship should be of the form  $T \propto l^a$  where the exponent  $a$  is found from your graph.

### Varying the gravitational field

The applet allows you to vary the gravitational field you cannot do in a real Earth-based laboratory!

- Select a value of  $l$ .
- Choose a new value of  $g$ .
- Find  $T$  by timing a known number of oscillations.
- Change  $g$ , keeping  $l$  constant, and find the new  $T$
- Continue until you have at least five sets of results.
- Enter your results into a spreadsheet.
- Plot a graph of  $\log T$  against  $\log g$ .
- Find the gradient of the graph.
- Hence obtain a relationship that shows how  $T$  varies with  $g$

Your relationship should be of the form  $T \propto g^b$  where the exponent  $a$  is found from your graph

## **Pendulum oscillations**

Combine your two expressions to give a single expression connecting  $T$ ,  $l$  and  $g$  in the form:

$$T = kl^a g^b$$

Use one of your sets of 'measurements' to determine the value of the constant  $k$ .

Use your complete formula to predict the period of a pendulum of length = 2.8 m in a gravitational field  $g = 9.8 \text{ N kg}^{-1}$

Check your prediction using the 'virtual pendulum'.

## **Practical advice**

Students will need Internet access. No other apparatus is needed. Use of a spreadsheet to record and graph results may also be appropriate.

## TAP 304- 3: Pendulum

The pendulum is often used as an example of a simple harmonic oscillator. Consider an ideal pendulum consisting of a point mass,  $m$ , in a gravitational field,  $g$ , at the end of a light string of length  $l$ . The pendulum is displaced to one side of its rest position such that the string makes an angle to the vertical of  $\phi$ .

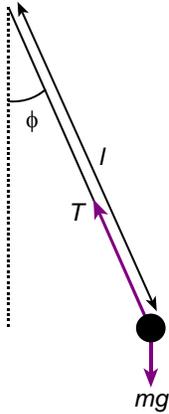
1. Draw a force diagram for the pendulum in this position.
2. In what direction does the resultant force act?
3. Derive an expression for the resultant force in terms of  $m$ ,  $g$  and  $\phi$ .
4. How does the force vary with angular displacement?
5. Why is the pendulum a good example of simple harmonic motion?  
Under what conditions the pendulum could not be used as a good example of simple harmonic motion?

## Practical advice

These are interesting but challenging questions, to stretch more able students.

## Answers and worked solutions

1. Force diagram must show weight ( $mg$ ) and the tension in the string, e.g.:



2. The resultant force must act tangential to the arc traced out by the swinging mass and hence perpendicular to the string.
3. Restoring force = component of weight perpendicular to the string  
 $= mg \cos(90 - \phi)$   
 $= mg \sin \phi.$
4. Force is a function of the sine of angular displacement.
5. This only works for simple harmonic motion where  $\phi$  is small so  $\sin \phi$  is considered to be the same as  $\phi$ . The pendulum is not a simple harmonic oscillator when the difference between  $\sin \phi$  and  $\phi$  is too large.