TAP 304- 3: Pendulum

The pendulum is often used as an example of a simple harmonic oscillator. Consider an ideal pendulum consisting of a point mass, *m*, in a gravitational field, *g,* at the end of a light string of length *l*. The pendulum is displaced to one side of its rest position such that the string makes an angle to the vertical of ϕ.

1. Draw a force diagram for the pendulum in this position.

2. In what direction does the resultant force act?

3. Derive an expression for the resultant force in terms of *m, g* and ϕ.

4. How does the force vary with angular displacement?

5. Why is the pendulum a good example of simple harmonic motion?

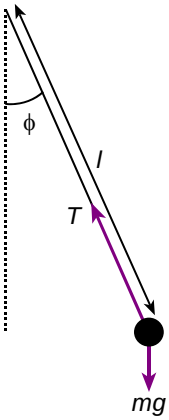
   Under what conditions the pendulum could not be used as a good example of simple harmonic motion?

Practical advice

These are interesting but challenging questions, to stretch more able students.

Answers and worked solutions

1. Force diagram must show weight (*mg*) and the tension in the string, e.g.:



2. The resultant force must act tangential to the arc traced out by the swinging mass and hence perpendicular to the string.

3. Restoring force = component of weight perpendicular to the string

   $= mg \cos(90 - \phi)$

   $= mg \sin\phi$.

4. Force is a function of the sine of angular displacement.

5. This only works for simple harmonic motion where ϕ is small so sin ϕ is considered to be the same as ϕ. The pendulum is not a simple harmonic oscillator when the difference between sin ϕ and ϕ is too large.