

## TAP 303- 4: Harmonic oscillators

A U-tube manometer is half-filled with water. The liquid levels are displaced so that one side is higher than the other. The water is then left free to oscillate from side to side. The tube has a cross section area of  $1\text{ cm}^2$  and the initial displacement is  $0.1\text{ m}$  from the rest position, which is  $0.5\text{ m}$  above the middle of the bottom of the tube. The total length of tube filled with water is  $1.1\text{ m}$ .

$$g = 10\text{ N kg}^{-1}$$

1. Using the relationship: pressure = density  $\times$  g  $\times$  depth, calculate the pressure at the bottom of the tube due to each of the unbalanced columns of water and hence the resultant force acting.  $g = 10\text{ m s}^{-2}$
2. How will the force vary as the levels move towards their rest positions?
3. Show that the restoring force is proportional to the displacement and hence that the resultant motion is simple harmonic.
4. Explain why the period remains constant as the oscillations die away due to friction.
5. How will the motion differ if mercury is used instead of water, with a density 13.6 times greater?

### Bobbing floats

Archimedes' principle states that the up thrust acting on a body immersed partly or wholly in a fluid is equal to the weight of fluid displaced. A cylindrical fishing float is  $15\text{ cm}$  long, with an

average cross section of  $3 \text{ cm}^2$ . It is made of polystyrene and has negligible mass. A lead weight of 30 g is attached to the bottom of the float using a thin nylon monofilament line.

$$g = 10 \text{ N kg}^{-1}$$

6. Assuming the weight and line to be of negligible volume, calculate how far will the float sink into the water?
  
  
  
  
  
  
  
  
  
  
7. A fish pulls on the baited hook further down the line and pulls the float down a further 3 cm before letting go. Calculate the resultant force on the float as the fish lets go.
  
  
  
  
  
  
  
  
  
  
8. Show that the restoring force is proportional to the distance that the float is pulled below its rest level.
  
  
  
  
  
  
  
  
  
  
9. If the float is lifted slightly by the angler and dropped, show that the force is again proportional to the distance above the rest position.

### Practical advice

These are interesting but challenging questions, to stretch more able students.

### Answers and worked solutions

1.

$$p = \rho gh = 1000 \text{ kg m}^{-3} \times 10 \text{ N kg}^{-1} \times 0.4 \text{ m} = 4000 \text{ Pa}$$

$$p = 1000 \text{ kg m}^{-3} \times 10 \text{ N kg}^{-1} \times 0.6 \text{ m} = 6000 \text{ Pa}.$$

Resultant force

$$F = pA = 2000 \text{ Pa} \times 0.0001 \text{ m}^2 = 0.2 \text{ N.}$$

2. Force is proportional to displacement.
3.  $F = [\rho g(0.5 + x)] - [\rho g(0.5 - x)]A = 2x\rho gA$   
i.e.  $F$  is proportional to  $x$ .
4. The term equivalent to the 'spring constant' and the mass remain the same so the frequency is unchanged.
5. The restoring force will be greater due to the larger pressure difference, caused by the greater density and hence proportionately greater effective spring constant. The mass to be moved will also be greater in proportion to the density. As a result the two effectively cancel and so the time period is unchanged, if we consider the motion to be pure SHM.
6. The up thrust must support 30 g, so the up thrust is 0.3 N.  
The float sinks until 0.3 N of water is displaced. Length of float submerged =  $h$ .  
 $\rho Vg = 0.3 \text{ N}$   
 $1000 \text{ kg m}^{-3} \times 3.0 \times 10^{-4} \text{ m}^2 \times h \times 10 \text{ N kg}^{-1} = 0.3 \text{ N}$   
 $h = 0.1 \text{ m}$
7. Additional volume displaced =  $0.03 \text{ m} \times 3 \times 10^{-4} \text{ m}^2 = 9 \times 10^{-6} \text{ m}^3$   
up thrust =  $V\rho g = 9 \times 10^{-6} \text{ m}^3 \times 1000 \text{ kg m}^{-3} \times 10 \text{ N kg}^{-1} = 0.09 \text{ N}$
8.  $F = \text{density} \times \text{area} \times g \times \text{additional length submerged}$ , therefore the force is proportional to the additional length submerged.
9. Lifting the float involves the same calculation, with the decrease in up thrust being calculated, which is then the resultant force downwards.

### External references

This activity is taken from Advancing Physics chapter 10, 190S