Episode 303: Mass-spring systems

This episode and the next focus on practical SHM systems.

Summary

Discussion: Hooke's law leads to SHM. (15 minutes)

Student experiment: Testing the relationship T = 2π√ (m/k). (30 minutes)

Student activity: Using a computer model. (20 minutes)

Discussion: Modelling in Physics. (10 minutes)

Worked example: Applying the relationship T = 2π√ (m/k). (10 minutes)

Student questions: Calculations involving T = 2π√ (m/k). (30 minutes)

Discussion:
Hooke's law leads to SHM

If your specification requires it, here is where you can derive the expression for the period of a mass-spring system. We have $\omega = 2\pi f = \sqrt{(F/mx)}$ as the requirement for SHM, and $F = kx$ from Hooke's law.

It is easiest to deal with a horizontal mass-spring system first (because you can ignore gravity).

Student experiment:
Testing the relationship T = 2π√ (m/k).

Students can test the relationship $T = 2\pi\sqrt{(m/k)}$ for a mass-spring system. (Note that this expression is independent of g.)

Students may find that there is a systematic error, caused by the finite mass of the spring. Try modifying the simple theory to take into account the mass of the spring m_S:

$T = 2\pi\ [(m + m_S)/k]^{1/2}$

TAP 303-1: Loaded spring oscillator
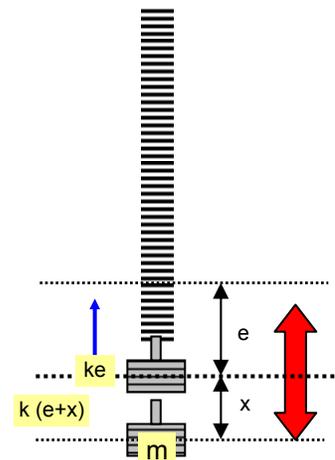
Student activity:
Using a computer model

They can also use a computer model.

TAP 303-2: Oscillating freely

and the model

TAP 303-3: Modelling springs and masses

Discussion:

Modelling in physics

The study of SHM may well be the first occasion that students meet detailed mathematical modelling. It may be worth spelling out for them what is happening. There is a physical behaviour we want to understand. First we simplify the actual situation to an idealised *physical model* by making assumptions, e.g. no 'friction', pendulum strings or springs that have no mass, etc. Then we make a *mathematical model* to represent the physical model. The mathematical model is then analysed ('solved') and has to be interpreted in terms of the physical model. Experiments try to mirror the physical model but they cannot do this exactly (e.g. make a pendulum string as light as possible while still being strong enough to support the 'bob'). So care is needed when comparing the theory with experimental measurements.

Worked example:

Applying the relationship T = $2\pi\sqrt{(m/k)}$

A vibrating atom in a solid can be modelled as a mass *m* between two tensioned springs, the springs representing the interatomic forces.

For typical interatomic forces k = 60 N m^{-1}

Mass of an atom (Na in NaCl) ~ 3.8×10^{-26} kg

Estimate the natural vibration frequency of atoms.

$f = 1/T = (1/2\pi) [2k/m]^{1/2}$

f ~ 9×10^{12} Hz, which is in the IR region of the electromagnetic spectrum. We will return to atomic vibrations when discussing resonance.

Student questions:

Calculations involving T = $2\pi\sqrt{(m/k)}$.

These questions reinforce basic ideas about SHM.

TAP 303-4: Harmonic oscillators.

TAP 303-1: Loaded spring oscillator

Context

Here you are asked to make careful measurements to check the effect of changing the mass and spring constant on the period of an oscillation.

You will need

- ✓ retort stand, boss and clamp and G-clamp to fix stand to bench
- ✓ steel spring
- ✓ mass hangers with slotted masses, 100 g
- ✓ hand held stopwatch

Linking the period and the mass

1. Set up spring and mass so that vertical oscillations can be measured.
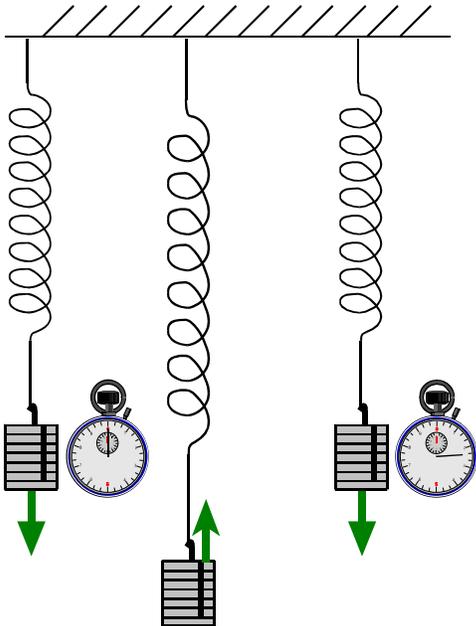
2. Decide how best to measure the time for one oscillation accurately – this is the period T.

3. Choose a range of masses so that the period varies significantly.

4. Make a number of measurements, taking care not to exceed the elastic limit of the spring, until you have about six pairs of mass-period data.

   Theory predicts that

   $$T = 2\pi \sqrt{\frac{m}{k}}$$

for the loaded spring oscillator.

5. Calculate the square of the period T^2 for each mass m, and draw an appropriate graph to check whether T^2 is proportional to m.

6. Calculate the gradient of your graph and hence find a value of the spring constant k.

7. To check do a simple Hooke's law experiment with your spring to check this value.

You have checked

1. The theoretical predictions relating the period of a loaded spring oscillator to the mass:

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Practical advice

This is a straightforward activity. It is assumed that the theory of the loaded spring oscillator has been developed and that this activity checks proportionality of T^2 to m. Students should concentrate on, and devise methods for, accurate measurement. Although some extendable springs appear to extend non-linearly for low added masses, this defect does not significantly reduce the possibility of students obtaining good results.

Social and human context

A light-hearted, but realistic, application of the result is considering baby bouncers. A baby's mass increases significantly over a period of 12 months, and students could be asked to calculate approximately (given that the bouncer is not a standard loaded spring oscillator) how this would affect the frequency of the oscillation.

An alternative application is that the relationship can be used to determine the mass of astronauts in free fall provided a 'two spring' system is used.

External reference

This activity is taken from Advancing Physics chapter 10, 260E

TAP 303- 2: Oscillating freely

Mass and spring oscillators

Many oscillators work on the interaction between springs and mass. Here you can look at the effect of these two quantities on the frequency of the oscillation. You will also probably do this in the laboratory, so you might use this software activity in a review session, or to exemplify your findings.
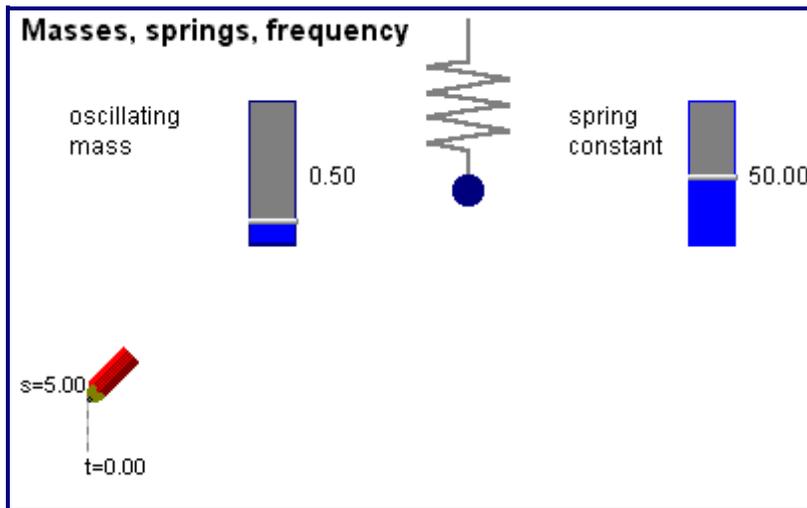
You will need

- ✓ computer running Modellus
- ✓ a Modellus model:

> TAP 303-3: Modelling springs and masses

Using the model

1.   Launch and run this model. Can you give a qualitative account of the motion?

Masses, springs, frequency

oscillating mass   0.50

spring constant   50.00

s=5.00

t=0.00

2.   What do you expect to happen to the motion when you double the mass? Can you explain why you expect this to happen? Try it and see, using the slider on the model to alter the mass.

3.   What do you expect to happen to the motion when you double the spring constant? Try to account for your expectations, and then use the slider on the model to alter the spring constant.

4.   If you have not already done so in the laboratory, collect a set of readings that show the relationship between the frequency (= 1 / period) and the mass. Then repeat for frequency and spring constant.

5.   Process these two sets of data to show the relationships graphically.

You have

1. Thought carefully about the dynamics of oscillators – how the forces and masses produce the resulting motions.

2. Found quantitative relationships between mass and frequency and between spring constant and frequency.

Practical advice

The activity emphasises F ∝ –x as the condition for SHM, and the relationship:

$$f \propto \sqrt{\frac{k}{m}}$$

This could be used after the free oscillator has been introduced. It will supplement work done in the laboratory in describing the motion, both in relating all the kinematic variables and in relating the characteristics of the oscillation to the dynamic variables. It could make a useful homework exercise. Alternatively you could demonstrate some of the features of the model, using it to introduce the topic. If so, a real system should be demonstrated also. At some stage the students should analyse real data.

This model is deliberately simple, and probably should not be used to replace laboratory work on this topic. It may, however, form a useful focus for discussion or private study. The chance to play the motion back step by step, talking through the changes to understand the dynamics, relating this to the average time for a complete oscillation, and to interact with a range of masses and spring constants should not be missed.

More confident students, or those with more time to spend here, could adapt the model to form a presentation, adding vectors to the animation, and perhaps slowing it down by making the time steps finer grained, to form a tool to help them explain the relationship between the quantities.

Alternative approaches

This model could be introduced much later when a lot of practical experience has been gained and students know about $(k/m)^{1/2}$, as you may choose to base an introduction entirely on laboratory work.

A web based JAVA Applet could also be used:
http://www.her.itesm.mx/academia/profesional/cursos/fisica_2000/FisicaII/PHYSENGL/springpendulum.htm

Social and human context

This step-by-step understanding, in which every change is linked to a prior sufficient cause, is central to the theme of the clockwork Universe and Laplace's thought: 'All the effects of nature are only the mathematical consequences of a small number of immutable laws'. Mass and spring oscillators can be used to model everything from car suspensions to molecular vibrations. It is important that students get a feel for the physics involved. In this activity, a model is employed to highlight some of the important physical ideas involved in studying simple oscillating systems.

Modellus

Modellus is available as a FREE download from http://phoenix.sce.fct.unl.pt/modellus/ along with other sample files and the user manual

External reference

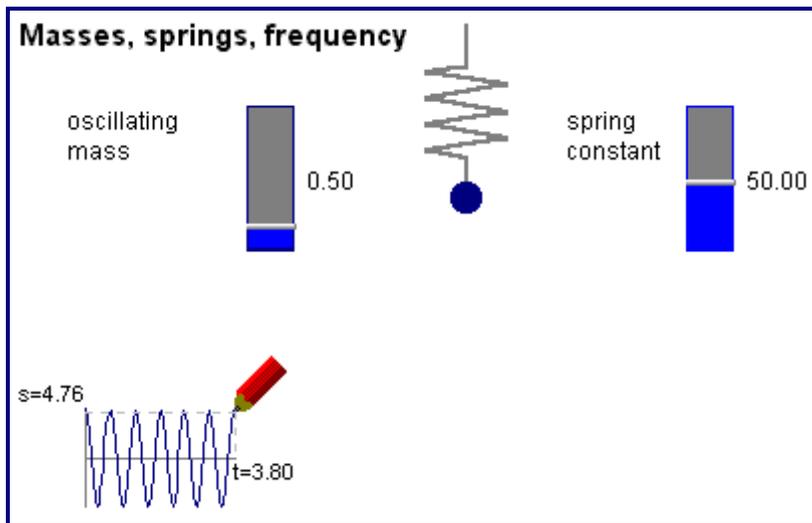This activity is taken from Advancing Physics chapter 10, 250s

TAP 303- 3: Modelling springs and masses

This file is provided for use with:

TAP 303-2: Oscillating freely

A Modellus model to look at the relationship between f, k and m

This model allows you to alter the spring constant and mass of an oscillator, looking at changes in the motion.

   The Modellus model is below

100039f1.mdl

Practical advice

This model looks at the relationship:

$$f \propto \sqrt{\frac{k}{m}}$$

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External reference

This activity is taken from Advancing Physics chapter 10, 130L

TAP 303- 4: Harmonic oscillators

A U-tube manometer is half-filled with water. The liquid levels are displaced so that one side is higher than the other. The water is then left free to oscillate from side to side. The tube has a cross section area of 1cm^2 and the initial displacement is 0.1 m from the rest position, which is 0.5 m above the middle of the bottom of the tube. The total length of tube filled with water is 1.1 m.

g = 10 N kg^{-1}

1.   Using the relationship: pressure = density × g × depth, calculate the pressure at the bottom of the tube due to each of the unbalanced columns of water and hence the resultant force acting. g = 10 m s^{-2}

2.   How will the force vary as the levels move towards their rest positions?

3.   Show that the restoring force is proportional to the displacement and hence that the resultant motion is simple harmonic.

4.   Explain why the period remains constant as the oscillations die away due to friction.

5.   How will the motion differ if mercury is used instead of water, with a density 13.6 times greater?

Bobbing floats

Archimedes' principle states that the up thrust acting on a body immersed partly or wholly in a fluid is equal to the weight of fluid displaced. A cylindrical fishing float is 15 cm long, with an average cross section of 3 cm^2. It is made of polystyrene and has negligible mass. A lead weight of 30 g is attached to the bottom of the float using a thin nylon monofilament line.

g = 10 N kg^{-1}

6.   Assuming the weight and line to be of negligible volume, calculate how far will the float sink into the water?

7.   A fish pulls on the baited hook further down the line and pulls the float down a further 3 cm before letting go. Calculate the resultant force on the float as the fish lets go.

8.   Show that the restoring force is proportional to the distance that the float is pulled below its rest level.

9.   If the float is lifted slightly by the angler and dropped, show that the force is again proportional to the distance above the rest position.

Practical advice

These are interesting but challenging questions, to stretch more able students.

Answers and worked solutions

1.

$$p = \rho g h = 1000 \text{ kg m}^{-3} \times 10 \text{ N kg}^{-1} \times 0.4 \text{ m} = 4000 \text{ Pa}$$
$$p = 1000 \text{ kg m}^{-3} \times 10 \text{ N kg} \times 0.6 \text{ m} = 6000 \text{ Pa.}$$

Resultant force

$$F = pA = 2000 \text{ Pa} \times 0.0001 \text{ m}^2 = 0.2 \text{ N.}$$

2. Force is proportional to displacement.

3. $F = [\rho g(0.5 + x)] - [\rho g(0.5 - x)]A = 2x\rho gA$

   i.e. F is proportional to x.

4. The term equivalent to the 'spring constant' and the mass remain the same so the frequency is unchanged.

5. The restoring force will be greater due to the larger pressure difference, caused by the greater density and hence proportionately greater effective spring constant. The mass to be moved will also be greater in proportion to the density. As a result the two effectively cancel and sp the time period is unchanged, if we consider the motion to be pure SHM.

6. The up thrust must support 30 g, so the up thrust is 0.3 N.

   The float sinks until 0.3 N of water is displaced. Length of float submerged = h.

$$\rho V g = 0.3 \text{ N}$$
$$1000 \text{ kg m}^{-3} \times 3.0 \times 10^{-4} \text{ m}^2 \times h \times 10 \text{ N kg}^{-1} = 0.3 \text{ N}$$
$$h = 0.1 \text{ m}$$

7. Additional volume displaced = $0.03 \text{ m} \times 3 \times 10^{-4} \text{ m}^2 = 9 \times 10^{-6} \text{ m}^3$

   up thrust = $V\rho g = 9 \times 10^{-6} \text{ m}^3 \times 1000 \text{ kg m}^{-3} \times 10 \text{ N kg}^{-1} = 0.09 \text{ N}$

8. F = density $\times$ area $\times g \times$ additional length submerged, therefore the force is proportional to the additional length submerged.

9. Lifting the float involves the same calculation, with the decrease in up thrust being calculated, which is then the resultant force downwards.

External references

This activity is taken from Advancing Physics chapter 10, 190S