

## Episode 315: The inverse square law

This episode considers the ways in which the intensity of radiation decreases with distance from the source.

### Summary

**Demonstration + discussion: radiation spreading out, and an analogy. (20 minutes)**

**Worked example: Thinking in proportions. (5 minutes)**

**Student experiment: Estimating the power output of the Sun. (15 minutes)**

**Student questions: Comparing lamps. (30 minutes)**

### Demonstration + discussion:

#### Radiation spreading out and an analogy

Set up three simple demonstrations:

- Bright lamp and phototransistor or other light meter
- Microwave source and detector
- Gamma ray source and Geiger counter

Show that the intensity of each type of radiation decreases with distance from the source.

(If students have already studied radioactivity, they may be aware of the inverse square law for gamma radiation.)

Why does the intensity decrease with distance? Try to give two general reasons. (Absorption and spreading out.)

Radiation spreading out radially covers a bigger and bigger area, proportional to  $r^2$ , so its intensity decreases as  $1/r^2$ . It can help to think of a 'toast-buttering gun'. The gun can butter a single slice of toast at a distance of 1 m. How can it butter four slices simultaneously? (Place them in a  $2 \times 2$  array at a distance of 2 m.) How thick will the butter be? ( $1/4$  of original thickness.) Extend this to a  $3 \times 3$  array at 3 m, and so on.

Point out that the intensity of radiation can be measured in watts per square metre ( $\text{W m}^{-2}$ ).

### Worked example:

#### Thinking in proportions

A cobalt 60 source gives a gamma dose rate of  $160 \mu\text{Sv h}^{-1}$  at 1.0 m away. At what distance will the dose rate be  $40 \mu\text{Sv h}^{-1}$ ?

Answer: If the intensity has gone down by a factor of 4, the distance away must have doubled to 2.0 m. Or by  $I = k/d^2$  so  $I_1/I_2 = d_2^2/d_1^2$ .

**Student experiment:**

**Estimating the power output of the Sun**

Students can make measurements to estimate the power output of the Sun, making use of the inverse square law. Note that there is a good opportunity here to discuss the validity of the answer obtained.

TAP 315-1: Summer Sun remembered

**Student questions:**

**Comparing lamps**

Some calculations.

TAP 315-2: Comparing intensities for lamps

## TAP 315-1: Summer Sun remembered

### Measuring brightness, hoping to measure distance

Comparing energies incident on the face allows an estimate of the distance of the Sun.

#### You will need

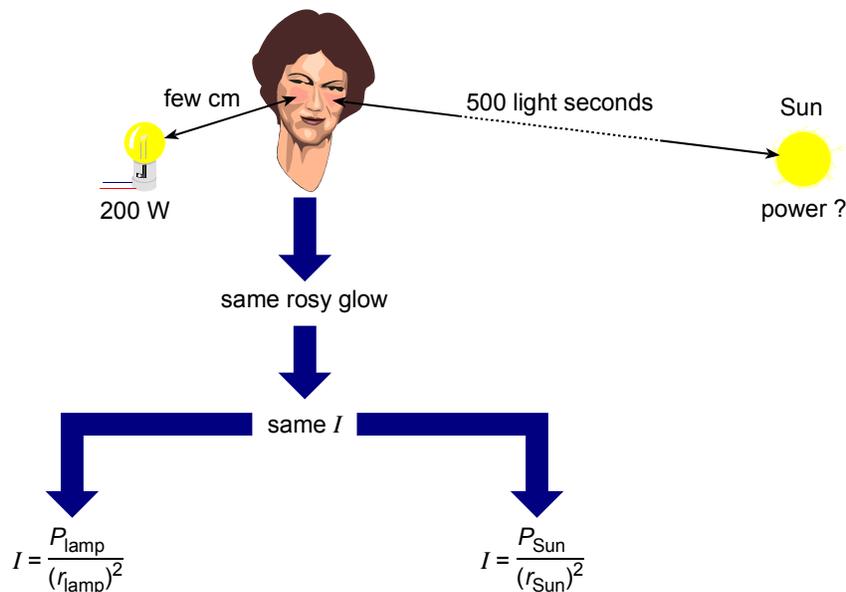
- ✓ high-powered lamp, perhaps 200 W (use a safety-pattern batten lamp holder wired to plug directly into a 13 A socket)
- ✓ ruler
- ✓ partner

	<b>Safety</b> The second student is important, as one cheek will have to get quite close to the 200 W lamp.
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#### What to do

Close your eyes, imagine lying on a beach and remember the summer Sun on your face. How hot did it feel?

Now bring your face up close to a high-power lamp – take care! A partner should watch you carefully, so that you do not touch the lamp. Aim to feel the same warmth that you remember in the summer Sun. Record the distance to the lamp and the power of the lamp.



Can you use these data to suggest the power output of the Sun? Remember that the Sun is about 500 light-seconds away.

### You have

1. Used the  $1 / r^2$  relationship to relate power outputs.

### Practical advice

The incident power per square metre is identical, and so application of the inverse square law will relate the power outputs of the lamp and Sun to the distances of the two.

A calculation can also show that the Sun's radiation at the Earth is of the order  $1 \text{ kW m}^{-2}$ .



### Safety

The second student is important, as one cheek will have to get quite close to the 200 W lamp.

### Social and human context

The power output of the Sun might usefully be compared with the power output of all human power stations.

### External reference

This activity is taken from Advancing Physics chapter 12, 50E

## TAP 315-2: Comparing intensities for lamps

### Comparing brightness

A low-power lamp at close range can often appear as bright as a high-power lamp much further away. Similar reasoning is used to calculate the brightness of stars.

### Two filament lamps

Here you compare two incandescent filament lamps; one marked 2.5 V, 0.3 A and a second 100 W, 240 V. Assume that approximately 20% of the power dissipated by the lamps results in visible photons being emitted uniformly, and that both are being run at normal brightness.

1. Estimate the brightness of the 100 W lamp from 3 metres away, which is the luminous flux density in  $\text{W m}^{-2}$
2. Calculate how close you must be to the 2.5 V lamp to see it as being the same flux density?

### Two floodlights

This question is about a 3 W bicycle headlight and a 300 W exterior security floodlight. Both the floodlight and the headlight have approximately the same shaped beam, a cone that covers approximately 1 / 10 of a sphere.

3. Estimate the flux density of the bicycle lamp from 20 metres away.
4. Your pupil has a diameter of approximately 0.01 m.  
How much light enters your eyeball per second?
5. The floodlight looks just as bright as the bicycle lamp.  
How far away is it?

## Practical advice

All these questions involve use of the inverse square law, sameness of beam shape being crucial in the questions about the floodlights.

## Answers and worked solutions

1. 20 W as light, spread over a sphere, of radius 3 m:

$$I = \frac{20 \text{ W}}{4 \times \pi \times r^2}$$

$$I = \frac{20 \text{ W}}{4 \times \pi \times 3 \times 3 \text{ m}^2}$$

$$I = 0.18 \text{ W m}^{-2}$$

2. Power as light = 2.5 V x 0.3 A x .02 = 0.15 W.  
This must spread over a sphere, to give a flux density of 0.19 W m<sup>-2</sup>.

$$I = \frac{P}{4\pi r^2}$$

$$r = \sqrt{\frac{P}{4\pi I}}$$

$$r = \sqrt{\frac{0.15 \text{ W}}{4 \times \pi \times 0.18 \text{ W m}^{-2}}}$$

$$r = 0.26 \text{ m.}$$

3. 3 W as light, focused onto 1 / 10 of sphere, of radius 20 m:

$$I = \frac{P}{4\pi r^2}$$

$$I = \frac{3 \text{ W}}{4 \times \pi \times (20 \text{ m})^2 \times 0.1} = 6 \text{ mW m}^{-2}$$

4. The pupil is catching light at 6 mW m<sup>-2</sup>:

$$P = I \times A$$

$$P = I \times \pi r^2$$

$$P = 6.0 \times 10^{-3} \text{ W m}^{-2} \times \pi \times 0.005^2 \text{ m}^2$$

$$P = 4.7 \times 10^{-7} \text{ W.}$$

5. The floodlight is 100 times more powerful than a cycle light:

$$I = \frac{P_f}{4\pi r_f^2} = \frac{P_b}{4\pi r_b^2}$$

$$\frac{300}{r_f^2} = \frac{3}{r_b^2}$$

$$\frac{r_f^2}{r_b^2} = \frac{300}{3}$$

$$\frac{r_f}{r_b} = 10$$

The floodlight is 10 times further away, 200 m.

### **External reference**

This activity is taken from Advancing Physics chapter 12, 40S