Tap 201-2: Displacement and vector addition

What this is about

This is a practical activity which needs to take place outside and which helps you to understand how vectors can be added.

You will need

- ✓ large space – a playground or playing field is ideal
- ✓ long tape measure or trundle wheel
- ✓ navigational compass

What to do:
Part 1

In any direction walk 40 m, in a straight line, and then turn right through a right angle, and walk 30 m in this new direction. Measure these distances accurately and note the start and finish positions. Now measure the distance between your starting point and finishing point. You should find this is 50 m. Why is this? The diagram below should help you to understand why.
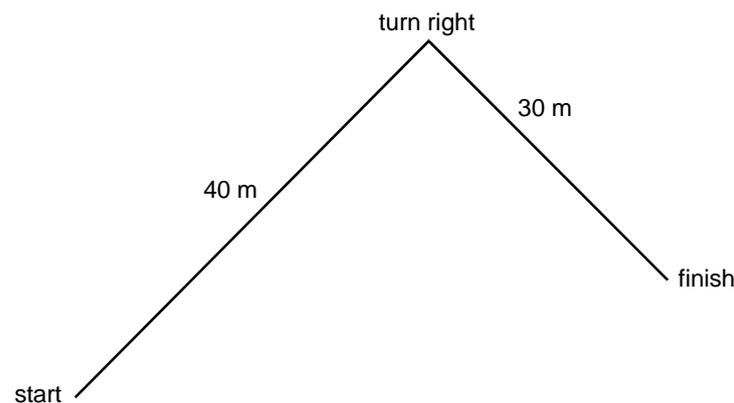

In the diagram the initial movements are vectors in that they have length and direction (though these are only relative to each other, **not** specific). Your resultant displacement is measured by the line which connects the start point to the finish point. If this is drawn, a triangle is formed, and as this has a right-angle Pythagoras' theorem can be used. The length of the resultant displacement is 50 m from the triangle. Your measured value includes errors of measurement, but should be nearly 50 m. The 40 m line and 30 m lines are displacement vectors. The resultant vector is found by completing the triangle of which they form two sides.

What to do:

Part 2

In the above case the original direction of motion was arbitrary. Next, include specific directions:

This time walk 40 m north, and then 30 m east. Measure the length of the resultant displacement AC, and think of a way of estimating the size of the angle marked θ.

B   30 m E   C

40 m N

θ
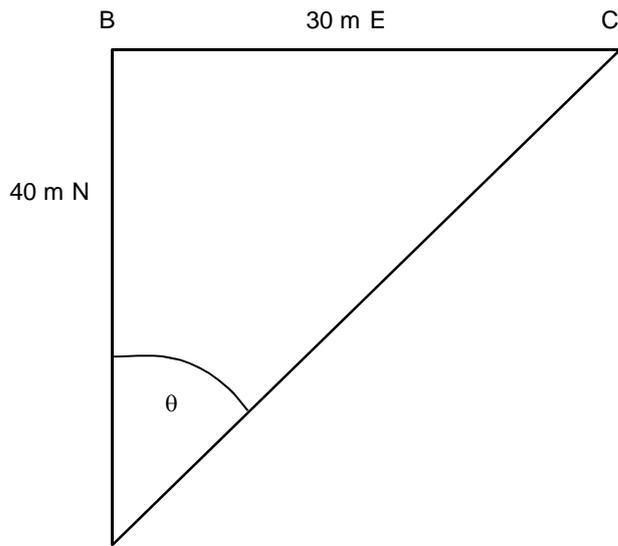
The resultant displacement should again be 50 m in length. It is said to be 50 m at an angle θ degrees east of north (written 50 m $θ^0$ E of N). Alternatively θ is written as a bearing. Bearings representing directions 0^0 to 360^0 measured clockwise from north are written as a three-digit number, so 045^0 is the direction NE and 315^0 is the direction NW.

You should have found that θ in your journey was about 037^0.

You should now be ready to show that this method of addition of two displacement vectors can be extended to deal with situations where a right angle is not included, and indeed can also be used for adding any pair of vectors.

What to do:
Part 3

Repeat your 40 m north walk, but then walk the 30 m north-east. Measure the length of the resultant displacement vector and its angle with the original northerly direction. Draw a diagram to scale so that you can verify your measurements.

Note that the scale diagram method has more general applications. Any pair of vectors, representing, for example, velocity, could be scaled in such a way and added by the triangle method.

What you have done

1.   Used Pythagoras' theorem to add two vectors at right angles.
2.   Used scale drawings to add vectors by completing a triangle to show the resultant.

Practical advice

A large space is needed and the means of measuring distances up to 75 m. Long tapes from PE departments and trundle wheels are possible, but the students will benefit from using pedometers. These are available from most outdoor activity shops at about £15. Alternatively, students could be asked to use bikes fitted with distance / speed computers. A navigational compass will be needed. Some means of noting results and sketching diagrams outside are required. In measuring angles outside, the compass can be used but a large board protractor may be helpful. Pythagoras' theorem, $a^2 + b^2 = c^2$, is required but the use of trigonometric ratios for calculating angles is optional, and related to the mathematical ability of the students.

Alternative approaches

Practically; Modellus models or other more lab scale activities can provide similar outcomes. You may be able to draw on kinaesthetic work using LOGO programming, perhaps studied in mathematics or find applets on the internet.

Social and human context

In days past, pacing out boundaries provided the only ways of mapping out land. Boundary stones were placed at significant places.

External references

This activity is taken from Advancing Physics Chapter 8, 110E