

## Episode 201: Scalars and vectors

Although many students will be familiar with the definition of vector and scalar quantities others may have glossed over these ideas. Such students may feel nervous about vector quantities and will find simple mathematical analysis quite daunting at the outset. This is particularly noticeable if this topic is taught early in the course. It is important that students get a 'feel' (literally!) for vectors alongside an understanding of their mathematical representation. It is worthwhile spending a little time on this introductory episode even if your specification does not make reference to it as it supports the work on moments and turning effects.

It is valuable to use practical work, computer simulations and pen-and-paper exercises to build student confidence in this topic.

### Summary

**Discussion: Define scalar and vector quantities; examples. (15 minutes)**

**Presentation and discussion: Adding vectors at right angles. (20 minutes)**

**Student experiment: Displacement and vector addition (20 minutes)**

**Worked examples: Adding perpendicular vectors. (20 minutes)**

**Student question: Practice for students. (15 minutes)**

### Discussion:

#### **Define scalar and vector quantities; examples.**

For some students this will be a revision session. Others may be less familiar with the concepts. Use question and answers to establish the knowledge of the group.

Start by defining the vectors as quantities with magnitude and direction. Compare these to scalar quantities (magnitude only). Use class discussion to develop a list of vector and scalar quantities (e.g. vectors: velocity, displacement, force; scalars: temperature, speed, distance, energy). It is useful to spend a little time on the difference between velocity and speed.

Discuss examples of simple, parallel vector addition. For example, a bullet fired from a moving train or a sled pulled across the snow against a constant frictional force. These easy examples build confidence in the basic principle. Extend this to anti-parallel vectors (i.e. vectors in opposite directions along same line; they subtract).

### **Demonstration + discussion:**

#### **Adding vectors at right angles**

This leads to a consideration of non-parallel vectors. Begin with vectors at right angles.

TAP 201-1: Adding velocities

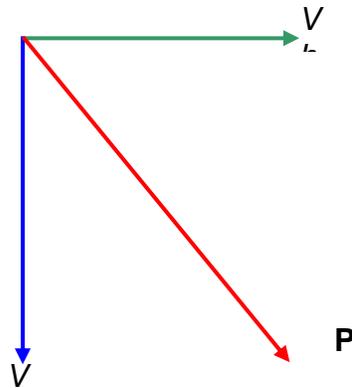
This demonstration takes a little time to set up and is over pretty quickly. However, it is worth the effort. You can perform the presentation with one student helper or you can act as a 'conductor' of two student volunteers.

Make clear that the ink trail is the result of adding two **perpendicular** velocities.

Measure the duration of the journey to establish the three velocities.

Repeat the presentation with a clean sheet of paper. This time have the 'horizontal' trolley moving at a new velocity to give a different gradient to the ink trail.

Draw a 'cleaned up' version of the results on a board or ask the students to do this in their notes. The diagram should include a scale so that direct measurements can be taken. You will get something like this:



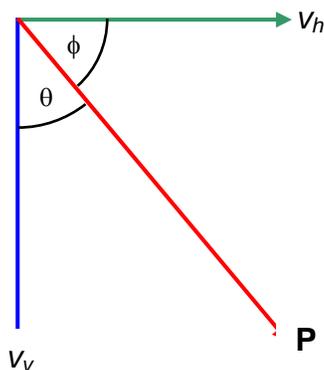
E.g. Scale: 1 cm = 2 cm s<sup>-1</sup>

It can be seen that the magnitude of **P** can be calculated using Pythagoras' theorem:

$$P^2 = v_h^2 + v_v^2$$

This can be checked against the results from the presentation. It can also be seen that a scale diagram can be used to predict the magnitude of **P** if the two horizontal and vertical components are known.

Simple trigonometry leads to the relationships  $v_y = P \cos \theta$  and  $v_h = P \cos \phi$



This can also be checked against the results from the presentation. This is the resolution of the velocity **P** into horizontal and vertical components. Further discussion will show that:

$$v_y = P \sin \phi$$

$$v_h = P \sin \theta$$

Students should note down these important relationships which will be used in the worked examples and student questions.

### Student experiment:

#### Displacement and vector addition

TAP 201-2: Displacement and vector addition

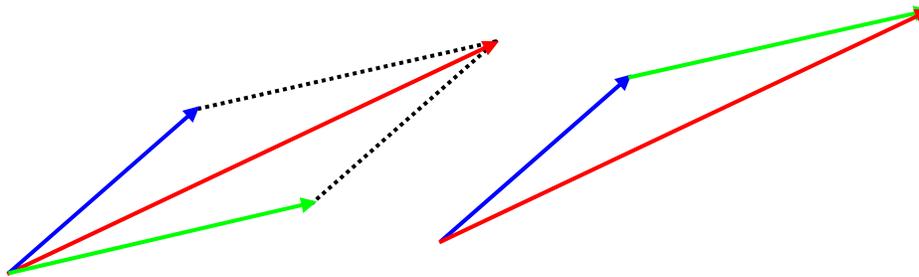
Your students can now put the theory to test, weather permitting! This activity begins with adding orthogonal vector displacements which will confirm the class discussions. The third activity sums displacements which are not at right angles. The simplest method of estimating the sum is to use a scale drawing – this may require explaining and extending to the general principle of the vector triangle. Students should be asked why Pythagoras cannot be applied to the third situation.

This should be considered as a quick activity to consolidate various ideas. Some students will believe that they know all about this; nevertheless, the practice is useful.

Make the addition of vectors by scale drawing explicit – a diagram such as the one below should be included in the discussion; in each case, the red arrow is the sum of the green and blue arrows.



(resourcefulphysics.org)



### Worked Examples:

#### Adding perpendicular vectors

TAP 201-3: Using the components of a vector

Use these examples to back up the work on vectors at right angles. The third part of the question is more difficult and shows the care needed in tackling such problems. The question is rather long and could be started in class as a discussion and left to finish for homework.

**Student question:**

**Practice for students**

TAP 201-4: Flying in a side wind.

This standard question will enable you to identify those students that have grasped the basic idea.

## TAP 201- 1: Adding velocities

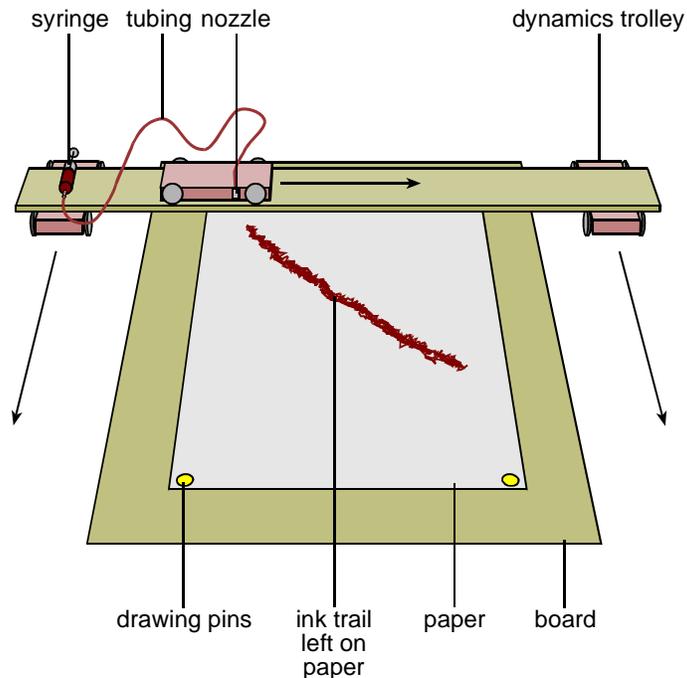
Velocities obey the vector addition law.

You will have seen many examples of vector addition by now. This activity is designed to show clearly that velocities follow vector addition laws. This law is sometimes referred to as geometric addition, in contrast to the arithmetic addition law you are familiar with.

### You will need

- ✓ three dynamics trolleys
- ✓ a small plank of wood or trolley runway, perhaps 1.25 m long
- ✓ A3 drawing paper, pins and a drawing board
- ✓ ink water, plastic syringe, tubing and Blue-tack
- ✓ ruler and protractor

### Obtaining a resultant velocity



Two dynamics trolleys have a bridge connected between them. The third trolley has a syringe filled with ink water and a length of suitable tubing connected to it with Blue-tack. A piece of drawing paper is placed beneath the apparatus as shown.

You need to work in pairs for this activity. One member of the team will push the connected trolleys along the bench or floor at a constant velocity. Some practice will be needed to achieve this. This velocity can be measured.

The other team member must produce a constant velocity along the runway. The two velocities must be produced simultaneously when results are being taken.

When you think you can work together to produce constant velocities, use the syringe to leave an ink-water trail as you move all the trolleys together. Experiment with different velocities and try to explain qualitatively the shape of the trails you record.

If you measure the velocities you are producing the trail can be used to check the vector law of addition.

On one of your ink-trail pictures, add calculations and diagrams – using Pythagoras' theorem and/or trigonometry – to show how the measured velocities of the trolleys lead to the resultant motion recorded by the ink trails. Your final product should be suitable for display on the laboratory wall.

## Interpretation

1. If you have measured the velocities of the trolleys then measuring the angle of the trail will allow you to cross check how well they fit the law of vector addition.
2. You should be able to verify that velocities obey the vector law of addition.
3. You might like to experiment with giving the trolley on the bridge a constant acceleration. This could be done using an incline. What shape of trail would you predict now?

## Practical advice

This activity is suitable for all abilities. The average student will enjoy showing a familiar law at work. Using the apparatus to produce sensible results will pose an interesting challenge to all students. Faster pupils might enjoy looking at paths produced when the bridge trolley accelerates as it moves forwards at a uniform velocity. This is good preparation for chapter 9. Of course this activity could be presented to the group as a demonstration with the teacher enlisting help from the students.

Qualitatively students should be able to account for the shape of the ink trail. Quantitative results are possible with care. There is opportunity to get students to use trigonometry to verify from the component velocities the angle of the ink trail on the paper.

Syringes and tubing are available from Phillip Harris and the bridge can be any suitable piece of wood. Here is a good use for old wooden dynamics trolleys!

## Alternative approaches

It may be easier to achieve a constant velocity for the two trolleys by using a Meccano crank with a long extension and two strings (one for each trolley). As the crank is turned steadily the trolleys advance at constant velocity.

The alternative to the above is the experiment using a motor to tow a cloth across the floor as a trolley moves across it. This idea was first introduced in *Physics for the Enquiring Mind* by Eric Rogers (Princeton University Press). We found his experiment difficult to get quantitative results from. However, it can be used to add vectors that are not at right angles.

## External references

This activity is taken from *Advancing Physics* Chapter 8, 210P

## Tap 201-2: Displacement and vector addition

### What this is about

This is a practical activity which needs to take place outside and which helps you to understand how vectors can be added.

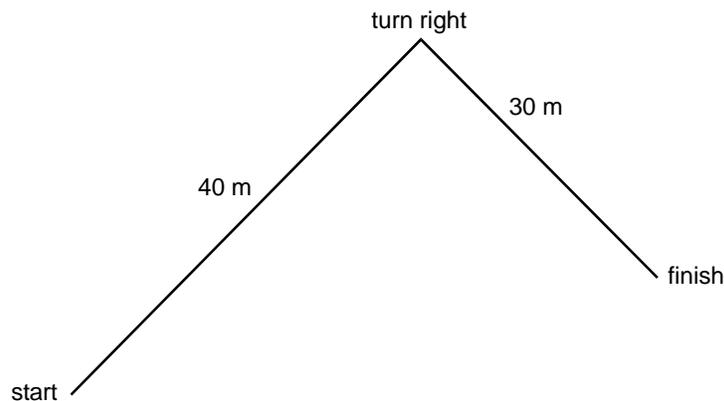
### You will need

- ✓ large space – a playground or playing field is ideal
- ✓ long tape measure or trundle wheel
- ✓ navigational compass

### What to do:

#### Part 1

In any direction walk 40 m, in a straight line, and then turn right through a right angle, and walk 30 m in this new direction. Measure these distances accurately and note the start and finish positions. Now measure the distance between your starting point and finishing point. You should find this is 50 m. Why is this? The diagram below should help you to understand why.



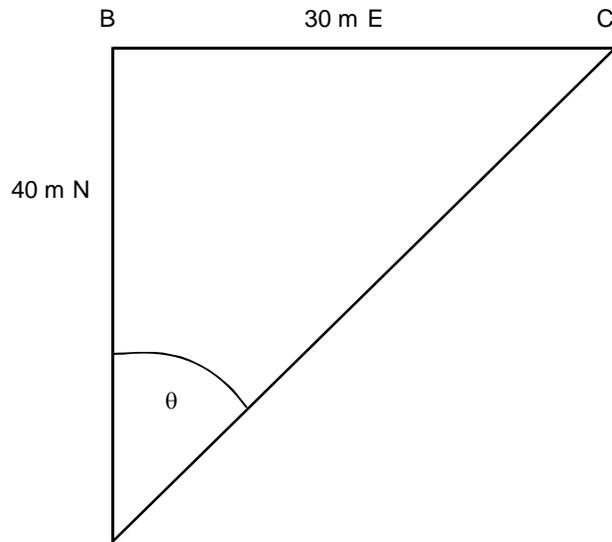
In the diagram the initial movements are vectors in that they have length and direction (though these are only relative to each other, **not** specific). Your resultant displacement is measured by the line which connects the start point to the finish point. If this is drawn, a triangle is formed, and as this has a right-angle Pythagoras' theorem can be used. The length of the resultant displacement is 50 m from the triangle. Your measured value includes errors of measurement, but should be nearly 50 m. The 40 m line and 30 m lines are displacement vectors. The resultant vector is found by completing the triangle of which they form two sides.

### What to do:

#### Part 2

In the above case the original direction of motion was arbitrary. Next, include specific directions:

This time walk 40 m north, and then 30 m east. Measure the length of the resultant displacement AC, and think of a way of estimating the size of the angle marked  $\theta$ .



The resultant displacement should again be 50 m in length. It is said to be 50 m at an angle  $\theta$  degrees east of north (written 50 m  $\theta^\circ$  E of N). Alternatively  $\theta$  is written as a bearing. Bearings representing directions  $0^\circ$  to  $360^\circ$  measured clockwise from north are written as a three-digit number, so  $045^\circ$  is the direction NE and  $315^\circ$  is the direction NW.

You should have found that  $\theta$  in your journey was about  $037^\circ$ .

You should now be ready to show that this method of addition of two displacement vectors can be extended to deal with situations where a right angle is not included, and indeed can also be used for adding any pair of vectors.

### What to do:

#### Part 3

Repeat your 40 m north walk, but then walk the 30 m north-east. Measure the length of the resultant displacement vector and its angle with the original northerly direction. Draw a diagram to scale so that you can verify your measurements.

Note that the scale diagram method has more general applications. Any pair of vectors, representing, for example, velocity, could be scaled in such a way and added by the triangle method.

### What you have done

1. Used Pythagoras' theorem to add two vectors at right angles.
2. Used scale drawings to add vectors by completing a triangle to show the resultant.

### **Practical advice**

A large space is needed and the means of measuring distances up to 75 m. Long tapes from PE departments and trundle wheels are possible, but the students will benefit from using pedometers. These are available from most outdoor activity shops at about £15. Alternatively, students could be asked to use bikes fitted with distance / speed computers. A navigational compass will be needed. Some means of noting results and sketching diagrams outside are required. In measuring angles outside, the compass can be used but a large board protractor may be helpful. Pythagoras' theorem,  $a^2 + b^2 = c^2$ , is required but the use of trigonometric ratios for calculating angles is optional, and related to the mathematical ability of the students.

### **Alternative approaches**

Practically; Modelling models or other more lab scale activities can provide similar outcomes. You may be able to draw on kinaesthetic work using LOGO programming, perhaps studied in mathematics or find applets on the internet.

### **Social and human context**

In days past, pacing out boundaries provided the only ways of mapping out land. Boundary stones were placed at significant places.

### **External references**

This activity is taken from Advancing Physics Chapter 8, 110E

## TAP 201- 3: Using the components of a vector

### Instructions

A vector quantity is a quantity with both magnitude (size) and direction. Sometimes it can be helpful to find the component (or effect) of a vector in a particular direction. In this activity, you will gain further practice in calculating vector components, using trigonometry, and with scale drawing.

This activity has two parts:

Part 1: Working out the component of a vector in a particular direction.

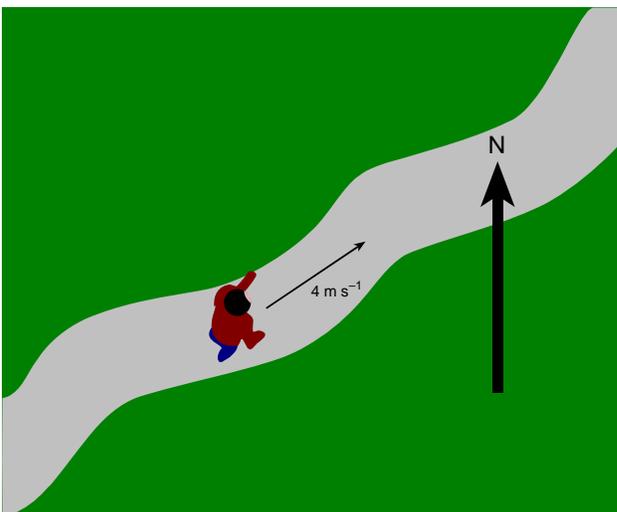
Part 2: Solving problems using components.

There are questions for you to try at the end of each part.

### Part 1: Working out the component of a vector in a particular direction

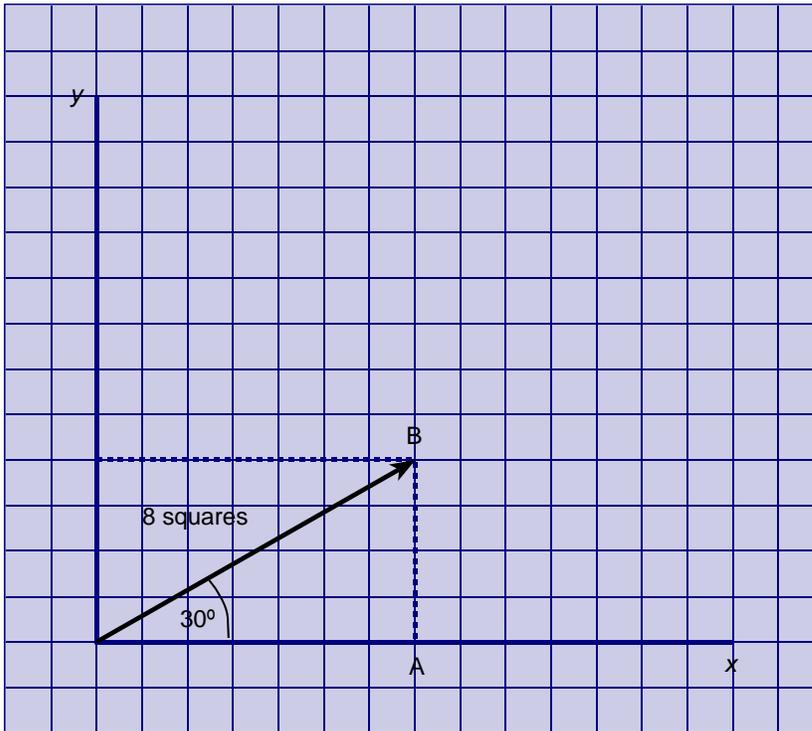
You will look at two ways of doing this: by drawing and by calculation (using trigonometry).

#### Finding components by drawing



You are walking at  $4.0 \text{ m s}^{-1}$  in a direction  $30^\circ \text{ N of E}$ . What is the component of your velocity in an easterly direction?

Look at the plan view above. Make a scale drawing of this on graph paper. The x-axis points east, the y-axis points north. You need to choose a suitable scale.



Here, the scale chosen is  $2 \text{ divisions} = 1 \text{ m s}^{-1}$ , so the arrow representing velocity is  $8.0 \text{ divisions}$  long. It is drawn at an angle of  $30^\circ$  to the x-axis, using a protractor to give the angle.

You need to find the component of velocity in the easterly direction. From the end of the velocity arrow, draw a line straight line down to the x-axis, point A.

Measure from the origin O to A. Convert this distance using the scale into  $\text{m s}^{-1}$ :

length of line =  $6.9 \text{ divisions}$

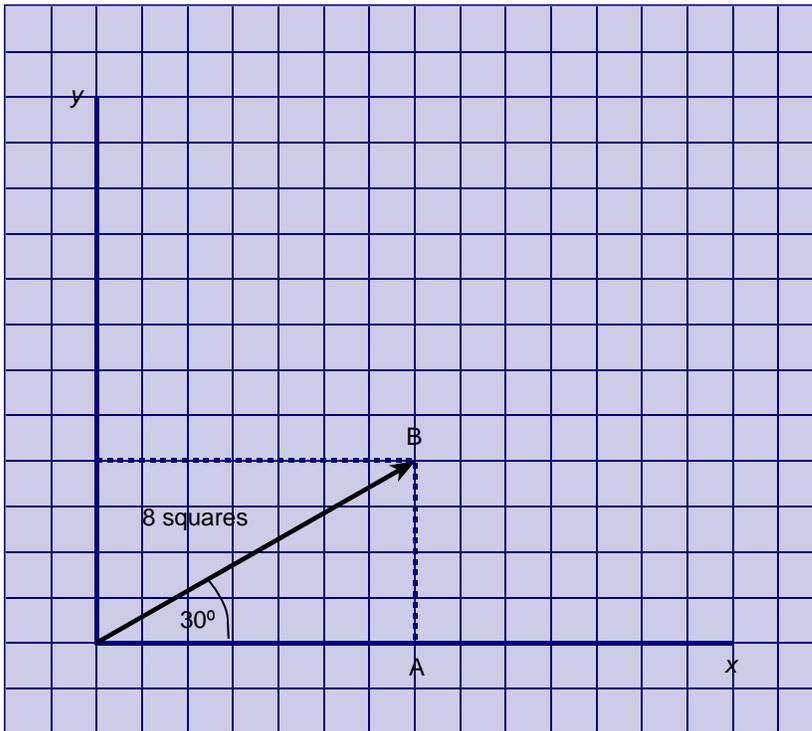
component of velocity towards the east =  $3.45 \text{ m s}^{-1}$

At the same time, you can see from the diagram that AB represents the component of the velocity towards the north. Its magnitude is  $2 \text{ m s}^{-1}$ .

This method is limited by the precision with which you can draw and measure. How precisely can you measure the angle? How precisely can you draw the velocity arrow? How accurately can you measure the length OA? All of these factors affect the precision of your final answer.

### Finding components by calculation

If you know some simple trigonometry, you may have realised how to calculate the component of velocity in the example above. Here is the diagram again:



You have a right-angled triangle OAB. You know the length of the long side (the hypotenuse) and you need to find the length of one of the other sides. They are related by the cosine of the angle AOB. Now:

$$\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

so you have:

$$\cos 30^\circ = \frac{OA}{OB} = \frac{OA}{4.0 \text{ m s}^{-1}}$$

Re-arranging gives:

$$OA = 4.0 \text{ m s}^{-1} \times \cos 30^\circ$$

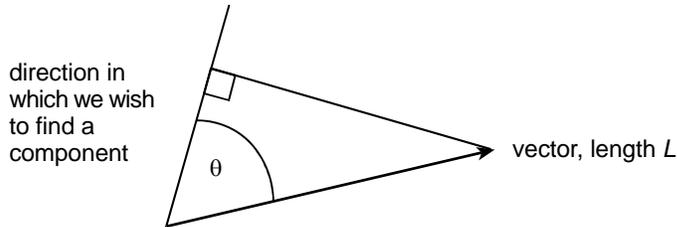
and calculation gives:

$$OA = 3.46 \text{ m s}^{-1}$$

This is the same answer as you found by scale drawing, but without the uncertainties introduced by drawing. Take care not to be misled by the apparent accuracy of this answer. (The calculator gave 3.464 1016... m s<sup>-1</sup>.) If your speed is given as 4.0 m s<sup>-1</sup>, you can only give the value of the component to two significant figures, i.e. 3.5 m s<sup>-1</sup>.

## Rule for finding the component of a vector in a particular direction

There is a general rule for finding the component of a vector in a particular direction. Look at the diagram below.



The length of the vector is  $L$ . Starting from the vector; find the angle  $\theta$  between it and the direction of interest. Then the component of  $L$  in this direction is  $L \cos \theta$ .

### Question 1

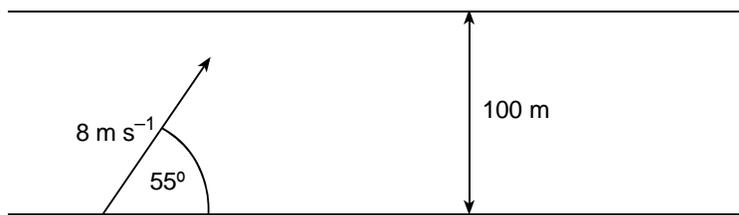
You are running at a speed of  $8 \text{ m s}^{-1}$  in a north-easterly direction (i.e. at  $45^\circ$  to both N and E). Find the components of your velocity in an easterly direction, and in a northerly direction, firstly by drawing, and then by calculation using trigonometry.

Explain why these two components have the same magnitude.

### Part 2:

#### Solving problems using components

You set off to run across an empty supermarket parking strip, 100 m wide. You set off at  $55^\circ$  to the verge, heading towards the entrance to the supermarket. Your speed is  $8 \text{ m s}^{-1}$ . How long will it take you to reach the far entrance?



Find the component of your velocity directly across the parking strip. The angle between your velocity and the direction of interest is  $35^\circ$  (why?). So:

$$\text{component of velocity across the parking strip} = v \times \cos \theta = 8 \text{ m s}^{-1} \times \cos 35^\circ = 6.55 \text{ m s}^{-1}.$$

Find the time it will take to travel 100 m at this speed:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{100 \text{ m}}{6.55 \text{ m s}^{-1}} = 15.3 \text{ s}.$$

You will reach the far side of the parking strip in a little more than 15 s.

Notice that calculating the time only involves the component of velocity directly **across** the parking strip. You can go on to calculate how far along the opposite side of the parking strip you will arrive.

Find the component of your velocity **along** the parking strip. The angle between your velocity and the direction of interest is  $55^\circ$ . So:

$$\text{component of velocity across the parking strip} = v \times \cos \theta = 8 \text{ m s}^{-1} \times \cos 55^\circ = 4.59 \text{ m s}^{-1}.$$

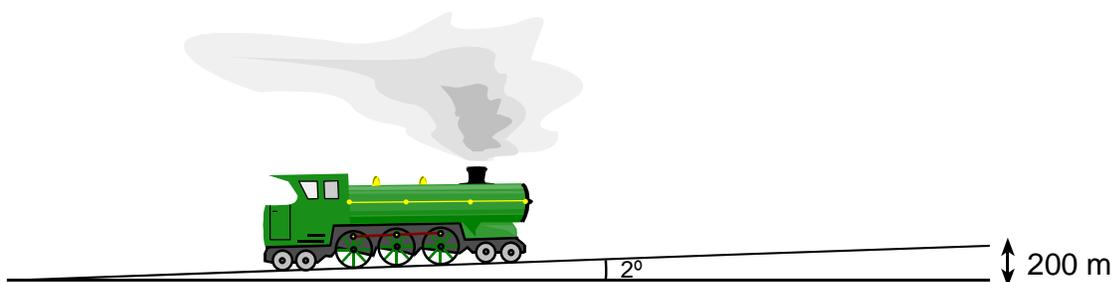
Find the distance travelled at this speed in 15.3 s:

$$\text{distance} = \text{speed} \times \text{time} = 4.59 \text{ m s}^{-1} \times 15.3 \text{ s} = 70.2 \text{ m}$$

So you will finish up about 70 m further along the parking strip. Notice that this calculation only involves the component of velocity *along* the parking strip.

## Question 2

A train is gradually travelling up a long gradient. The speed of the train is  $20 \text{ m s}^{-1}$  and the slope makes an angle of  $2^\circ$  with the horizontal. The summit is 200 m above the starting level. How long will it take to reach the summit?



## Practical advice

The text provides revision of the basic ideas about components and the questions provide some simple reinforcement. There are many similar questions elsewhere. The advantage of these questions is that a step-by-step method is set out and illustrated by a worked example and the questions then test the application of this method.

## Alternative approaches

Standard texts contain many examples similar to the ones here.

## Social and human context

Travel of all sorts provides appropriate contexts.

## Answers and worked solutions

1. As the velocity vector makes an angle of  $45^\circ$  with north, the component north is

$$8 \text{ m s}^{-1} \times \cos 45^\circ = 5.7 \text{ m s}^{-1}.$$

The component east can be calculated in the same way or by using the  $45^\circ$  angle between the vector and north to calculate the component east from the sine function:

$$8 \text{ m s}^{-1} \times \sin 45^\circ = 5.7 \text{ m s}^{-1}.$$

2. To find the time taken to travel to the top of the hill you need first to find the distance along the slope. From the diagram in the question, it is clear that  $\sin 2^\circ = 200 / \text{slope}$ , so that the length of the slope is:

$$\text{slope} = 200 / \sin 2^\circ = 5731 \text{ m}.$$

Then the time taken is:

$$\text{time taken} = \text{distance} / \text{speed} = 5731 \text{ m} / 20 \text{ m s}^{-1} = 4 \text{ minutes } 47 \text{ seconds}.$$

### External references

This activity is taken from Advancing Physics Chapter 8, 80S



## Practical advice

These questions reinforce ideas about vector addition and relative velocity. In addition, the connection between velocity and displacement is tested.

## Alternative approaches

There are many similar questions in standard texts.

## Social and human context

Aeroplane flight, boats crossing rivers, etc. All deal with vector components explicitly or implicitly.

## Hints

1. Velocity is a vector, and has both magnitude and direction.
2. How is velocity defined in terms of displacement?
3. To fly due north requires the bird not to move in an easterly direction. What must the east–west component of its velocity be?

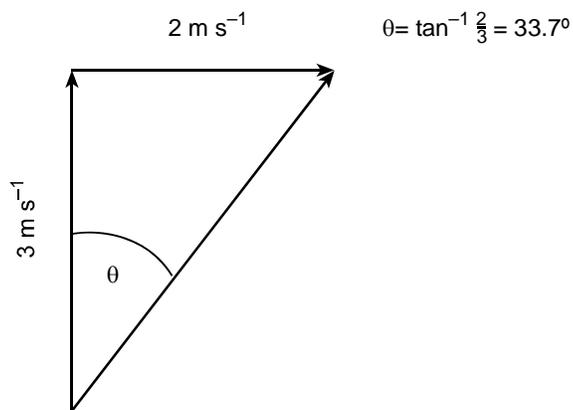
## Answers and worked solutions

1. The magnitude of the resultant velocity is found using Pythagoras' theorem as the two vectors to be added are at right angles.

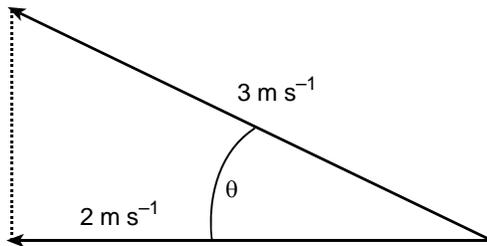
Hence: magnitude of velocity relative to the ground.

$$= \sqrt{3^2 + 2^2} \text{ m s}^{-1} = 3.6 \text{ m s}^{-1}.$$

The direction is found by drawing a vector diagram.



2. After 20 s, the displacement of the bird is found from displacement = velocity  $\times$  time. The direction of displacement is as for velocity if the velocity is constant. The magnitude of the displacement is simply  $3.6 \text{ m s}^{-1} \times 20 \text{ s} = 72 \text{ m}$ .
3. To fly due north, the bird has to fly in such a direction that it has a component of its velocity that cancels out the velocity of the wind. This means that it has to have a component of velocity equal to  $2 \text{ m s}^{-1}$  west. The diagram shows how this is done.



$$\theta = \cos^{-1} \frac{2}{3} = 48^\circ$$

$$\Rightarrow \text{bearing} = 270^\circ + 48^\circ = 318^\circ$$

### External references

This activity is taken from Advancing Physics Chapter 8, 70S