

TAP 201- 3: Using the components of a vector

Instructions

A vector quantity is a quantity with both magnitude (size) and direction. Sometimes it can be helpful to find the component (or effect) of a vector in a particular direction. In this activity, you will gain further practice in calculating vector components, using trigonometry, and with scale drawing.

This activity has two parts:

Part 1: Working out the component of a vector in a particular direction.

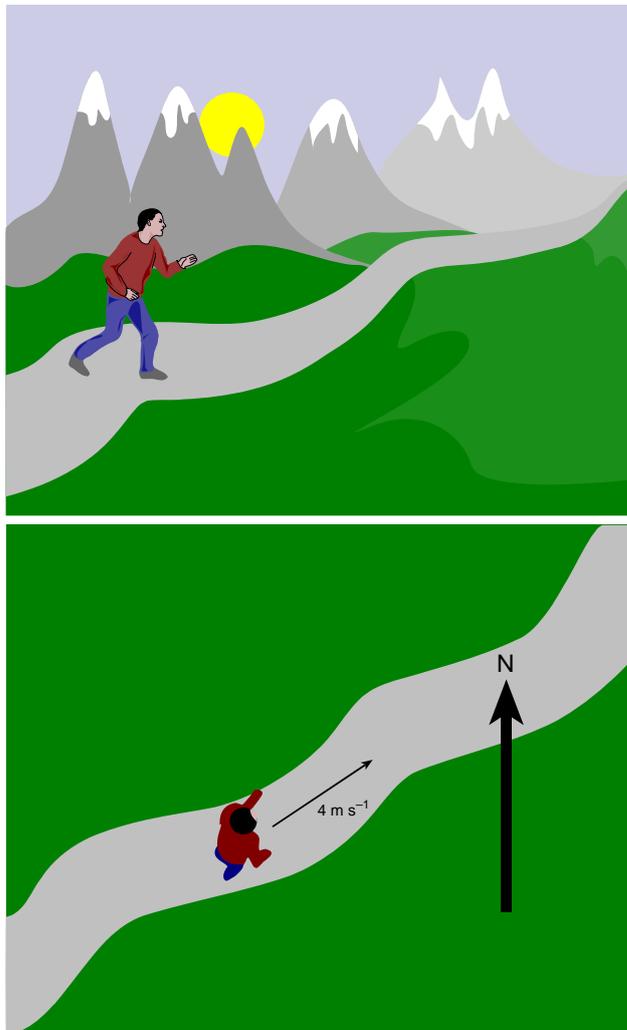
Part 2: Solving problems using components.

There are questions for you to try at the end of each part.

Part 1: Working out the component of a vector in a particular direction

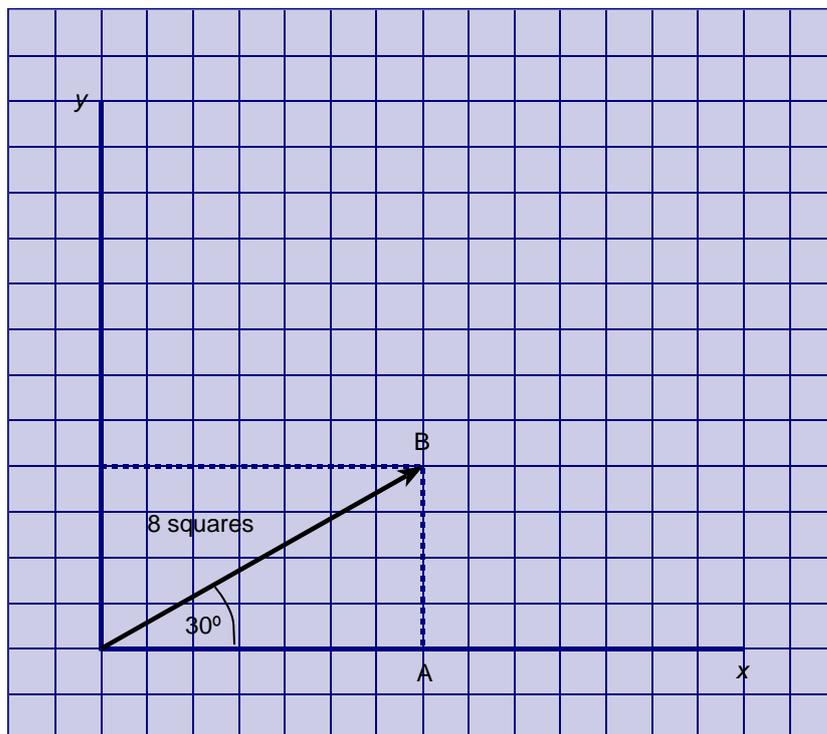
You will look at two ways of doing this: by drawing and by calculation (using trigonometry).

Finding components by drawing



You are walking at 4.0 m s^{-1} in a direction 30° N of E . What is the component of your velocity in an easterly direction?

Look at the plan view above. Make a scale drawing of this on graph paper. The x -axis points east, the y -axis points north. You need to choose a suitable scale.



Here, the scale chosen is 2 divisions = 1 m s^{-1} , so the arrow representing velocity is 8.0 divisions long. It is drawn at an angle of 30° to the x -axis, using a protractor to give the angle.

You need to find the component of velocity in the easterly direction. From the end of the velocity arrow, draw a line straight line down to the x -axis, point A.

Measure from the origin O to A. Convert this distance using the scale into m s^{-1} :

length of line = 6.9 divisions

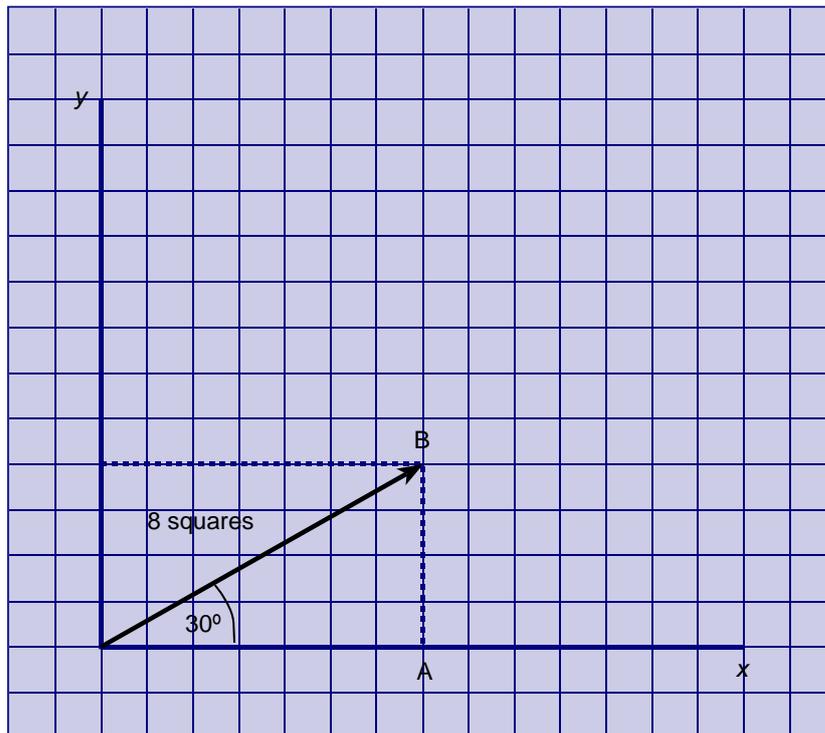
component of velocity towards the east = 3.45 m s^{-1}

At the same time, you can see from the diagram that AB represents the component of the velocity towards the north. Its magnitude is 2 m s^{-1} .

This method is limited by the precision with which you can draw and measure. How precisely can you measure the angle? How precisely can you draw the velocity arrow? How accurately can you measure the length OA? All of these factors affect the precision of your final answer.

Finding components by calculation

If you know some simple trigonometry, you may have realised how to calculate the component of velocity in the example above. Here is the diagram again:



You have a right-angled triangle OAB. You know the length of the long side (the hypotenuse) and you need to find the length of one of the other sides. They are related by the cosine of the angle AOB. Now:

$$\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

so you have:

$$\cos 30^\circ = \frac{OA}{OB} = \frac{OA}{4.0 \text{ m s}^{-1}}$$

Re-arranging gives:

$$OA = 4.0 \text{ m s}^{-1} \times \cos 30^\circ$$

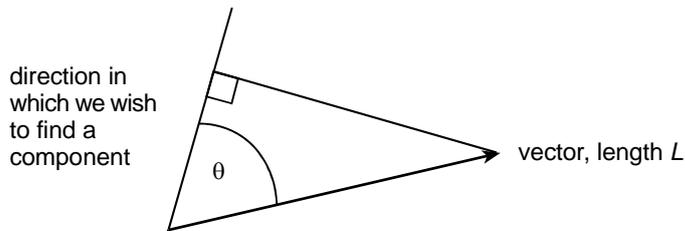
and calculation gives:

$$OA = 3.46 \text{ m s}^{-1}$$

This is the same answer as you found by scale drawing, but without the uncertainties introduced by drawing. Take care not to be misled by the apparent accuracy of this answer. (The calculator gave 3.464 1016... m s⁻¹.) If your speed is given as 4.0 m s⁻¹, you can only give the value of the component to two significant figures, i.e. 3.5 m s⁻¹.

Rule for finding the component of a vector in a particular direction

There is a general rule for finding the component of a vector in a particular direction. Look at the diagram below.



The length of the vector is L . Starting from the vector; find the angle θ between it and the direction of interest. Then the component of L in this direction is $L \cos \theta$.

Question 1

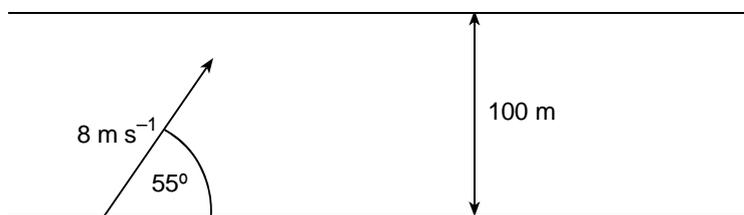
You are running at a speed of 8 m s^{-1} in a north-easterly direction (i.e. at 45° to both N and E). Find the components of your velocity in an easterly direction, and in a northerly direction, firstly by drawing, and then by calculation using trigonometry.

Explain why these two components have the same magnitude.

Part 2:

Solving problems using components

You set off to run across an empty supermarket parking strip, 100 m wide. You set off at 55° to the verge, heading towards the entrance to the supermarket. Your speed is 8 m s^{-1} . How long will it take you to reach the far entrance?



Find the component of your velocity directly across the parking strip. The angle between your velocity and the direction of interest is 35° (why?). So:

$$\text{component of velocity across the parking strip} = v \times \cos \theta = 8 \text{ m s}^{-1} \times \cos 35^\circ = 6.55 \text{ m s}^{-1}.$$

Find the time it will take to travel 100 m at this speed:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{100 \text{ m}}{6.55 \text{ m s}^{-1}} = 15.3 \text{ s}.$$

You will reach the far side of the parking strip in a little more than 15 s.

Notice that calculating the time only involves the component of velocity directly **across** the parking strip. You can go on to calculate how far along the opposite side of the parking strip you will arrive.

Find the component of your velocity **along** the parking strip. The angle between your velocity and the direction of interest is 55° . So:

$$\text{component of velocity across the parking strip} = v \times \cos \theta = 8 \text{ m s}^{-1} \times \cos 55^\circ = 4.59 \text{ m s}^{-1}.$$

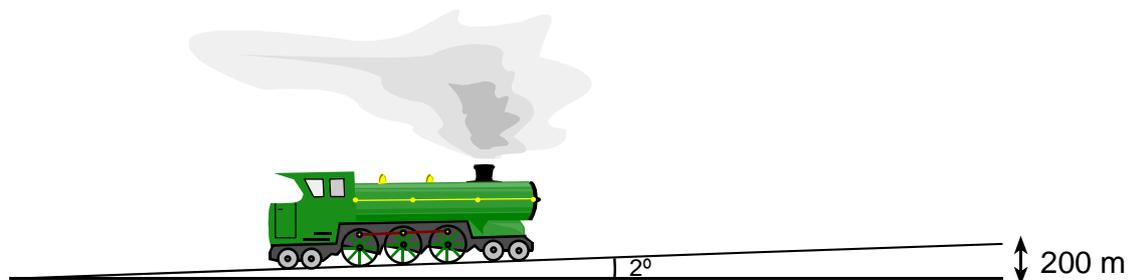
Find the distance travelled at this speed in 15.3 s:

$$\text{distance} = \text{speed} \times \text{time} = 4.59 \text{ m s}^{-1} \times 15.3 \text{ s} = 70.2 \text{ m}$$

So you will finish up about 70 m further along the parking strip. Notice that this calculation only involves the component of velocity *along* the parking strip.

Question 2

A train is gradually travelling up a long gradient. The speed of the train is 20 m s^{-1} and the slope makes an angle of 2° with the horizontal. The summit is 200 m above the starting level. How long will it take to reach the summit?



Practical advice

The text provides revision of the basic ideas about components and the questions provide some simple reinforcement. There are many similar questions elsewhere. The advantage of these questions is that a step-by-step method is set out and illustrated by a worked example and the questions then test the application of this method.

Alternative approaches

Standard texts contain many examples similar to the ones here.

Social and human context

Travel of all sorts provides appropriate contexts.

Answers and worked solutions

1. As the velocity vector makes an angle of 45° with north, the component north is

$$8 \text{ m s}^{-1} \times \cos 45^\circ = 5.7 \text{ m s}^{-1}.$$

The component east can be calculated in the same way or by using the 45° angle between the vector and north to calculate the component east from the sine function:

$$8 \text{ m s}^{-1} \times \sin 45^\circ = 5.7 \text{ m s}^{-1}.$$

2. To find the time taken to travel to the top of the hill you need first to find the distance along the slope. From the diagram in the question, it is clear that $\sin 2^\circ = 200 / \text{slope}$, so that the length of the slope is:

$$\text{slope} = 200 / \sin 2^\circ = 5731 \text{ m}.$$

Then the time taken is:

$$\text{time taken} = \text{distance} / \text{speed} = 5731 \text{ m} / 20 \text{ m s}^{-1} = 4 \text{ minutes } 47 \text{ seconds}.$$

External references

This activity is taken from Advancing Physics Chapter 8, 80S