# Episode 227: Hooke's law

This episode introduces the mechanical properties of materials: often an important consideration when choosing a material for a particular application. It focuses on the strength and stiffness of materials when subject to linear tensile or compressive forces (i.e. forces acting along a line in one dimension). Shearing forces (acting in a plane or two dimensions) or bulk forces (acting throughout a volume or in three dimensions) are not covered here.

The topic of Hooke's Law is likely to be a revision of existing knowledge.

## Summary

**Demonstration and discussion: Hooke's law. (20 minutes)**

**Student experiment: Stretching fibres. (30 minutes)**

**Demonstration: How stiffness depends on physical dimensions. (20 minutes)**

## Demonstration and discussion:

## Hooke's law

Start with a recap of Hooke's law. As a visual aid add 1 N weights (i.e. 100 g masses) to a suitable spring. If you also have a compression spring then so much the better. It should be apparent that

extension (or compression) $\Delta x \propto$ load

(Note: although many text books simply use *x* for extension when discussing Hooke's Law, it is helpful to use $\Delta x$ to avoid confusion later with the original length *x* when discussing the Young Modulus.)

Note that *a pair* of forces are involved when applying a tension or compression. If only one force acted on the sample, it would be 'unbalanced', which implies the sample would accelerate. If you suspend a spring from a support and hang a weight from it, the weight is one force; the other is the upward force provided by the support.

TAP 227-1: Strengths of some materials

Tension (in the sample) $F \propto$ extension

i.e. $F = k\,\Delta x$

This is a mathematical statement of **Hooke's law**, where *k* is the **stiffness**, equal to the tension per unit extension = $F/\Delta x$ Units N m$^{-1}$. (The opposite of stiffness is the **compliance**.)

The stiffness *k* can be found from the slope of the *linear* part of the $F$-$\Delta x$ graph – taking the slope averages all the individual data points.

## Student experiment:

## Stretching fibres

Students can perform some careful experiments to see if fibres (rather than springs) obey Hooke's law.

TAP 227-2: Tension and extension

*All* materials will show Hooke's Law behaviour 'up to a point'. This point is sometimes called the *elastic limit*. However, this is a simplification. Technically:

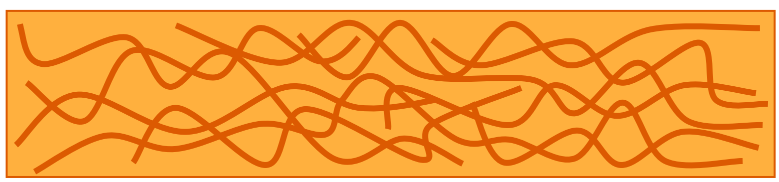
The linear part of the graph ends at the *limit of proportionality*.

The elastic part of the behaviour ends at the *elastic limit*.

These two points do not necessarily coincide. Behaviour is described as *plastic* beyond the elastic limit.

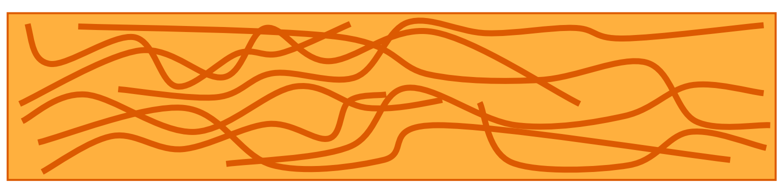
Loading many types of material (such as metals) simply stretches the bonds between the atoms from which it is made. The linear relationship between extension and load is simply a reflection of the fact that the bond force is varying linearly with the separation of the atoms. Plastic behaviour results from the permanent displacement of the atoms from their original positions.

Many polymers (e.g. rubber) show very large extensions – far too large to be explained by 'bond stretching'. Initially it is the uncoiling and/or straightening out of the long molecules that takes place. When they are all pulled straight, the sample gets much stiffer as bond stretching takes over.

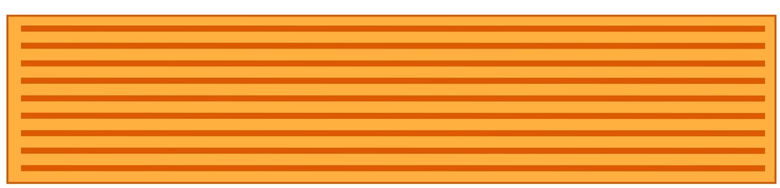


(resourcefulphysics.org)

Chain molecules tangled up



Chain molecules untangling



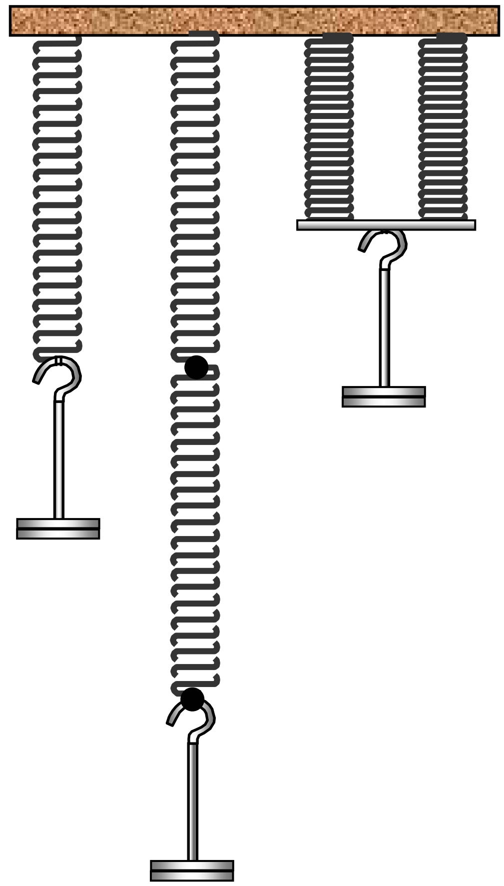
Chain molecules completely untangled

TAP 227-3: Explaining stiffness and elasticity

TAP 227-4: Plasticity in polythene

TAP 227-5: Elastic behaviour in rubber

## Demonstration



Stiffness *k* obviously depends upon the actual material; it also depends on the dimensions of the sample. Thicker samples stretch less per newton than thinner ones. Imagine two identical samples in parallel – twice the cross-sectional area *A* implies half the extension for given load, as each sample supports half the load, so *k* for the system as a whole is doubled.

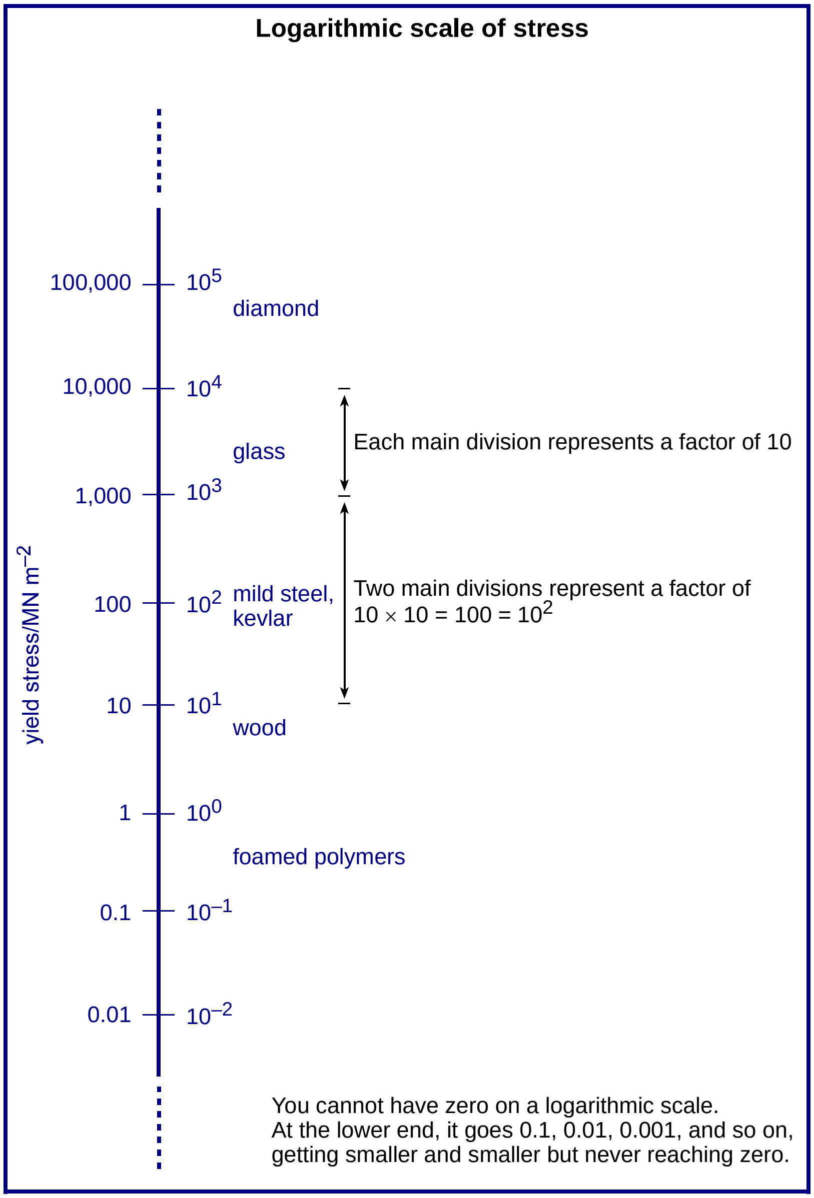
What about two identical samples in series? Both are subjected to the same load, thus each stretches the by the same amount, so the total stretch is doubled, so the stiffness for the system as a whole is halved.

Check the above conclusions in a demonstration using (identical) springs in parallel and in series.

TAP 227-6: More than one spring

# TAP 227-1: Strengths of some materials

A logarithmic scale shows how the strength of materials ranges over several orders of magnitude.



## Practical Advice

This provides an opportunity to develop ideas about powers of ten and logarithmic plots, through the work on strength of materials.

These points may be valuable to mention when discussing this scale:

The strength of materials varies over several orders of magnitude. It is easiest to show this on a logarithmic scale.

You cannot have zero on a logarithmic scale. At the lower end it goes 0.1, 0.01, 0.001, etc, getting smaller and smaller but never reaching zero.

## Alternative Approaches

The page could be printed out for students to add to their notes for revision.

## External References

This activity is taken from Advancing Physics Chapter 4, Display material 40O

# TAP 227- 2: Tension and extension

Find out how the extension of a fibre varies with tension and find the breaking strength of your sample.  Make sure you test the nylon sample as you will need your results in a question that follows.

## Apparatus

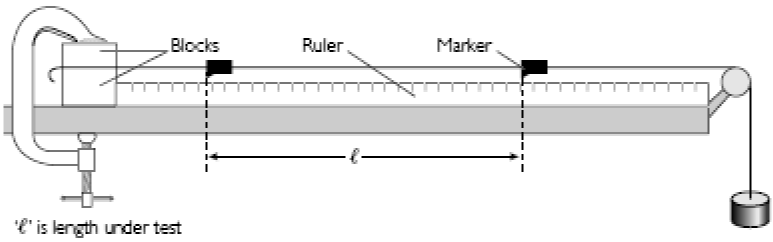
- ✓ selection of different reels of fibres – cotton, nylon (or other man-made polymer), string, wool
- ✓ lengths of rubber (ideally of differing cross-section, as sold by model shops)
- ✓ G-clamp
- ✓ small wooden blocks for gripping end of fibre
- ✓ clamped pulley
- ✓ several 1 kg fixed masses (could be filled sandbags)
- ✓ sets of 0.1 kg slotted masses on hangers
- ✓ sets of 10 g or 20 g slotted masses on hangers
- ✓ Sellotape
- ✓ card
- ✓ metre rule
- ✓ scissors
- ✓ eye protection
- ✓ soft 'landing platform', e.g. thick foam sheet or cardboard box containing packing material

**Safety**

**The load should not be too high off the floor, and there should be a suitable soft 'landing platform' (not your feet!) if (when) the sample breaks.**

**Use eye protection – beware of the fibre 'whipping back' if it breaks. Do not lean over the stretched fibre.**

## Set-up:




## What to do:

You will need to do a trial run to decide roughly on a suitable test length – the marker nearer the pulley may disappear over the edge! If the sample appears not to stretch very much, how could you improve matters? Take readings of load and extension as the load is gradually increased. ('Extension' means the total amount stretched from its original, natural, length – not the extra amount for each increase in load.) You will need a small (say 0.1 N) starting load just to take up the slack – ignore this in your load measurement (although it may well be insignificant anyway).

- If the load is reduced, is there any evidence of permanent stretch?

## Analysing the results

Decide on the best way of presenting your results in order to contrast the varying force–extension behaviour of the different samples.

Plot graphs of tension (*y*-axis) against extension. This may seem the 'wrong way round' since it is usual to put the independent variable (here tension) along the *x*-axis. Conventionally, results of experiments such as these have extension along the *x*-axis, since professional tensometers measure the force required to produce a controlled extension.

## Study the design of a climbing rope – why is it as it is?

Calculate the stiffness using a number of different *F*–*x* pairs for the same fibre. You are likely to obtain different values for the same fibre according to which point you select on the graph. Can you relate the stiffness, in general terms, to what the graph is doing and also to the nature of the fibre itself?

For many materials it is approximately true that at least over part of the range the stiffness is *constant*. To what extent is that true for your samples?

An 11.5 mm external diameter climbing rope is stated to have a breaking strength of 32 kN. Approximately how many nylon fibres are in the core if they have the same diameter as your sample?

## Practical Advice

This is a version of the ‘standard arrangement’ commonly used for testing metal wires. Even fine threads are surprisingly strong. To make this more ‘real’, student could test fine threads drawn from a climbing rope. Students left to their own devices may well end up with 4 or 5 kg hanging, with obvious hazards. It is possible to purchase stout bags with brass eyelets to be filled with 1 or 2 kg of sand for this type of testing – this is certainly preferable to delicately poised slotted masses.

## Hooke’s law

You might like to discuss with students the status of a physical ‘law’ that is only obeyed in some circumstances, as opposed to Newton’s laws of motion that are always obeyed.

If students are familiar with Ohm’s law from pre-16 level they should appreciate that its status is rather like that of Hooke’s law.

**Safety**

The load should not be too high off the floor, and there should be a suitable soft ‘landing platform’ (not your feet!) if (when) the sample breaks.

**Use eye protection** –beware of the fibre ‘whipping back’ if it breaks. Do not lean over the stretched fibre.

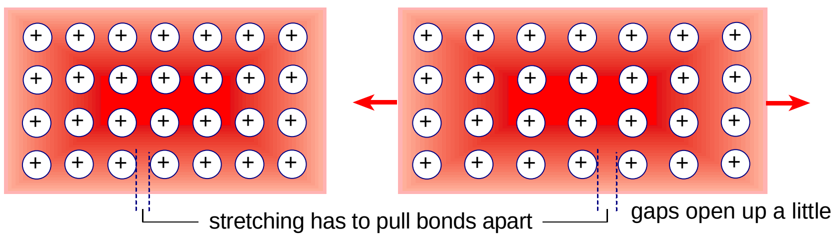
## External References

This activity is taken from Salters Horners Advanced Physics, Section HFS, Activity 18

# TAP 227- 3: Explaining stiffness and elasticity

**Explaining stiffness and elasticity**

**Metals**




a metal is an array of positive ions bonded by negative electron 'glue'

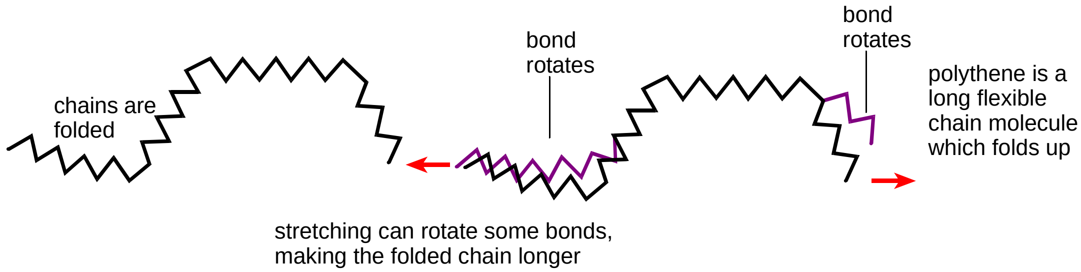
Elastic extensibility ~ 0.1%

Young modulus
~$10^{11}$ — $10^{12}$ Pa

Stretching a metal stretches bonds — but not much.

## Explaining stiffness and elasticity

**Polythene**




Elastic extensibility ~ 1%

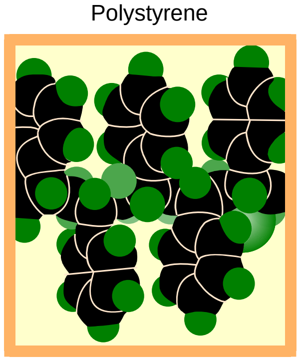
Young modulus
~$10^8$ — $10^9$ Pa

Stretching polythene rotates bonds

## Explaining stiffness and elasticity

**Stiffer polymers**

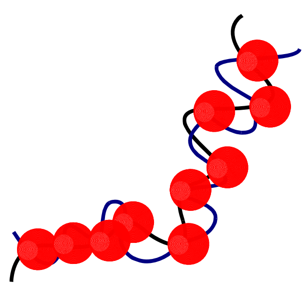
Polystyrene



Polystyrene has benzene rings sticking out sideways. They make chain rotations difficult.

Young modulus
~$10^9$ — $10^{10}$ Pa

Bakelite – a thermoset



Bakelite has massively cross-linked chains. The cross-links stop the chains from unfolding.

Young modulus
~$10^{10}$ Pa

## Practical Advice

That stretching polythene rotates bonds is worth talking about here.

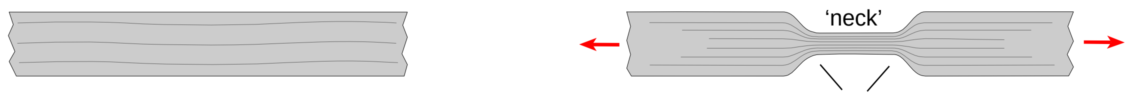
## External References

This activity is taken from Advancing Physics Chapter 5, 190O

# TAP 227- 4: Plasticity in polythene

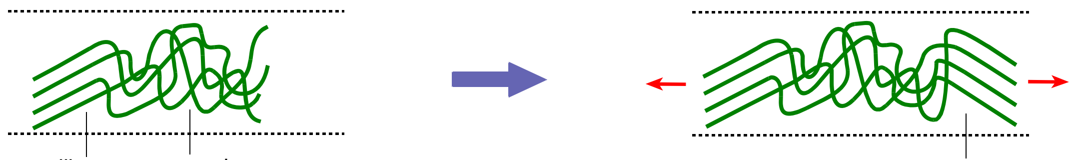
## Explaining plasticity

### Polythene




polythene strip 10 mm × 100 mm

thin crystalline strip 'pulled out of' wider region



crystalline amorphous

new crystalline region

Polythene is semi-crystalline. Think of polythene as like cooked spaghetti. In amorphous regions the chains fold randomly. In crystalline regions the chains line up.

When stretched plastically, the chains slip past each other. More of the material has lined-up chains. More of it is crystalline.

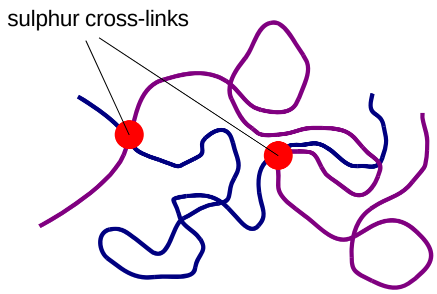
Plastic extensibility > 100%

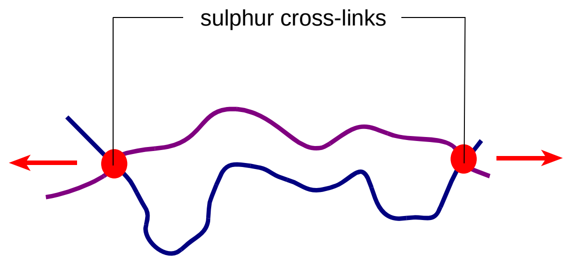
## External References

This activity is taken from Advancing Physics Chapter 5, 190O

# TAP 227- 5: Elastic behaviour in rubber

**Rubber**




In unstretched rubber, chains meander randomly between sulphur cross-links.

In stretched rubber, the chain bonds rotate, and chains follow straighter paths between cross-links. When let go, the chains fold up again and the rubber contracts.

Elastic extensibility > 100%

**Rubber stretches and contracts by chains uncoiling and coiling up again. Rubber is elastic, not plastic.**

## External References

This activity is taken from Advancing Physics Chapter 5, 210O

# TAP 227- 6: More than one spring

## Aim

To find out how the spring constant for systems of springs is related to that of a single spring.

## Requirements

- ✓ steel springs, tensile
- ✓ mass hangers with slotted masses, 100 g
- ✓ retort stand base, rod, boss and clamp
- ✓ short length of stiff wire (or similar) to combine springs in parallel
- ✓ G-clamp if the retort stand base is light weight

## Background

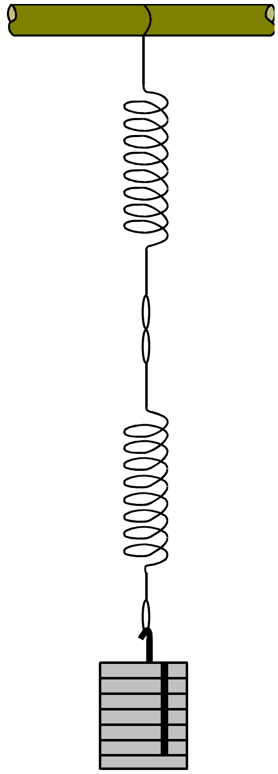
Each spring is described by Hooke's law, $F = k\Delta x$, for small extensions, $\Delta x$, of the spring.

## What to do

1. By plotting a graph of applied force, F, against extension, $\Delta x$, find the spring constant, k (restoring force per unit extension) for a single spring. Do not exceed the elastic limit of the spring.
2. Connect two springs in series. Find out how the spring constant, $k_s$, for this system is related to k for the single spring.

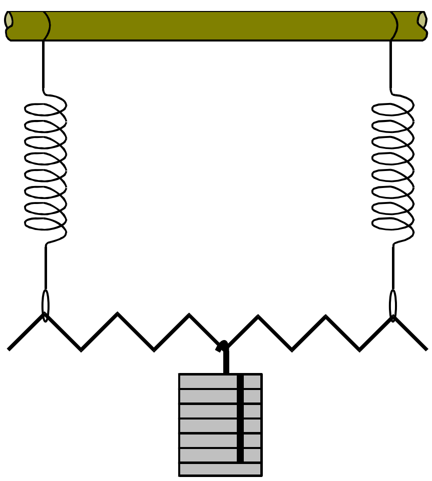
## Safety

A soft landing zone may be useful to prevent bruised toes.



springs in ‘series’

3. Now connect two springs in parallel so they jointly support the masses. Find the spring constant for this parallel arrangement, $k_p$, and find out how it is related to k for a single spring.



springs in ‘parallel’

## You can work out

1. A general rule relating the spring constant of a system of springs connected in series or parallel to the spring constant of a single spring.

## Practical advice

This activity leads into a consideration of the relationship between the microscopic structure of a material and the Young modulus.

With weaker students it will be necessary to carry out the experiment for them to 'see' how spring systems combine k. More able students may not need this concrete stage.

## Alternative approaches

You may not think you have time to do the whole of this. At the least it will be worth showing the class the spring model of a crystal and discussing how forces at the atomic level can be estimated by this kind of analysis.

## Social and human context

Thomas Young's interest in elasticity and the strength of materials arose from contemporary concerns about the design of two particular types of structure – bridges and warships. His lifetime (1773–1829) saw both the Napoleonic wars, in which British sea-power was crucial, and the building of four new bridges in central London: Waterloo, London, Southwark and Vauxhall, the first iron bridge to be built across the Thames.

## External References

This activity is taken from Advancing Physics Chapter 5, 140E