Episode 207: Projectile motion

This episode looks at the independence of vertical and horizontal motion. It concerns objects accelerating vertically when projected horizontally or vertically. The crucial concept is that vertical acceleration does not affect horizontal velocity. This explains all projectile motion. You can discuss why this is the case dynamically – but this is best left until later in the study of mechanics. Getting the basic concept across should be your priority. Similarly, make it clear that you are ignoring any effects of drag at this stage.

(Note that a projectile is an object which is initially projected by a force, but which then continues to move freely under the influence of gravity; a rocket which is firing its motors is not a projectile.)

Summary

Demonstration and discussion: Motion in a parabola. (10 minutes)

Demonstration: Monkey and hunter (10 minutes)

Demonstration: Pearls in air. (5 minutes)

Student investigation: Range of a projectile. (30 minutes)

Student experiment: Gravity and archery. (30 minutes)

Demonstration + Discussion:

Motion in a parabola

Here are two quick demonstrations showing motion in a parabola.

The first is a quick, fun demonstration that focuses the students' minds on parabolic motion. This is something we all 'know' but this episode is all about explaining the motion. Follow this with the 'diluted gravity' demonstration.

TAP 207-1: A thrown ball follows a parabolic path

TAP 207-2: Diluted gravity - projectile paths

Having successfully obtained a parabola the following tasks can be used to move the students' understanding forward:

Describe the motion, as precisely as possible, in words. (No hand waving!)

If this proves difficult try breaking up the motion into horizontal and vertical components. What is happening to the vertical velocity? (It's decreasing, then changing direction and increasing, i.e. vertical acceleration.) What about the horizontal velocity? (It's constant.)

Further pointers: Ignoring air resistance, is anything resisting the horizontal motion? (No!)

Will the acceleration due to gravity be different for a horizontally moving object? (No, again!)

You can use *Multimedia Motion* to generate a graph of projectile motion which clearly shows the independence of horizontal and vertical velocities.

The discussion should develop the ideas of independence of horizontal and vertical motion and uniform horizontal *velocity* and uniform vertical *acceleration*. Hence, horizontal and vertical displacements are given by:

$s_h = v_h t$ and sv = $u_v\, t + 1/2\, a\, t^{2.}$ In many cases u_v is zero.

Demonstration:

Monkey and hunter

The idea that vertical and horizontal motions can be considered separately is demonstrated in a dramatic fashion in the classic 'monkey and hunter' experiment.

You need to set this up in advance and check that it is working. When it does work it is a superb

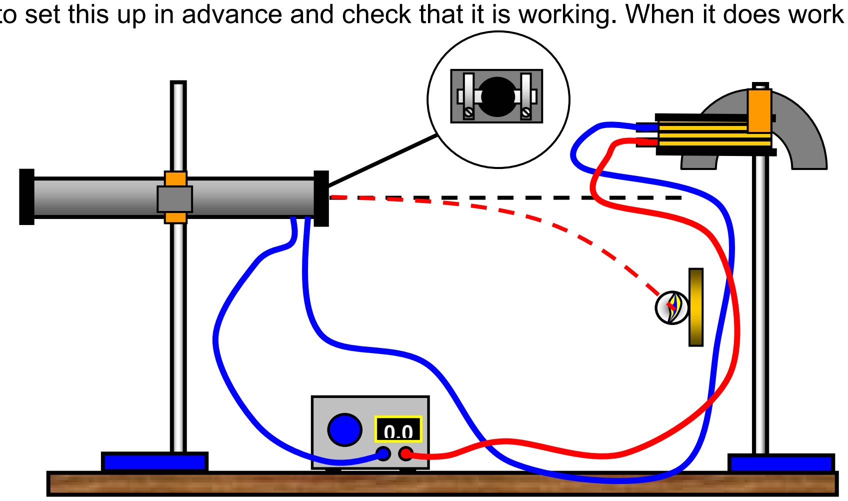

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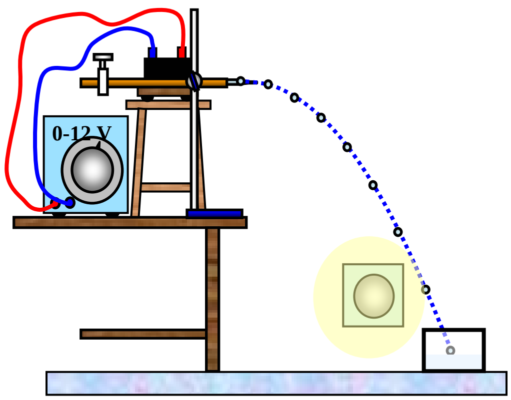
illustration. Questioning should focus on why this demonstrates independence of horizontal and vertical motion – both objects have fallen the same distance in the same time.

TAP 207-3: Mid-air collisions

Demonstration:

Pearls in air

Water droplets also follow a parabolic path in the air. This gives another clear demonstration of the effect. Again, concentrate on the explanation in terms of independence of horizontal and vertical motions.



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TAP 207-4: Pearls in air

Student investigation

Here is an interesting approach to projectile motion in which students fire a marble towards a target. This gives students, working independently or in pairs, an opportunity to design a simple experiment that will give them practice using the SUVAT equations.

TAP 207-5: Build and test a marble launcher

The students should begin by considering vertical motion. If they set their launcher x metres above the sand pit, the time the ball will be in the air can be found from:

$s_v = u_v t + 1/2\ a\ t^2$ where $s_v = x$ and $u_v = 0$; hence $t = \sqrt{(2x/g)}$.

From the horizontal range, the horizontal velocity can be calculated. This value can be used to predict the range when the height above the pit is changed to $2x$, $3x$, $4x$ and so on.

A table can be constructed with the following headings:

Height / m	Calculated time in air / s	Predicted range / m	Measured range / m

This can be completed for homework – it gives useful practice in SUVAT. Graphs can be constructed of predicted and measured ranges against height. Students should comment on the comparison between predicted and measured ranges.

Student experiment:

Gravity and archery

Keen students can extend the investigation to consider the effect of gravity in the sport of archery.

TAP 207-6: Challenges in target archery

TAP 207-1 A thrown ball follows a parabolic path

Demonstration

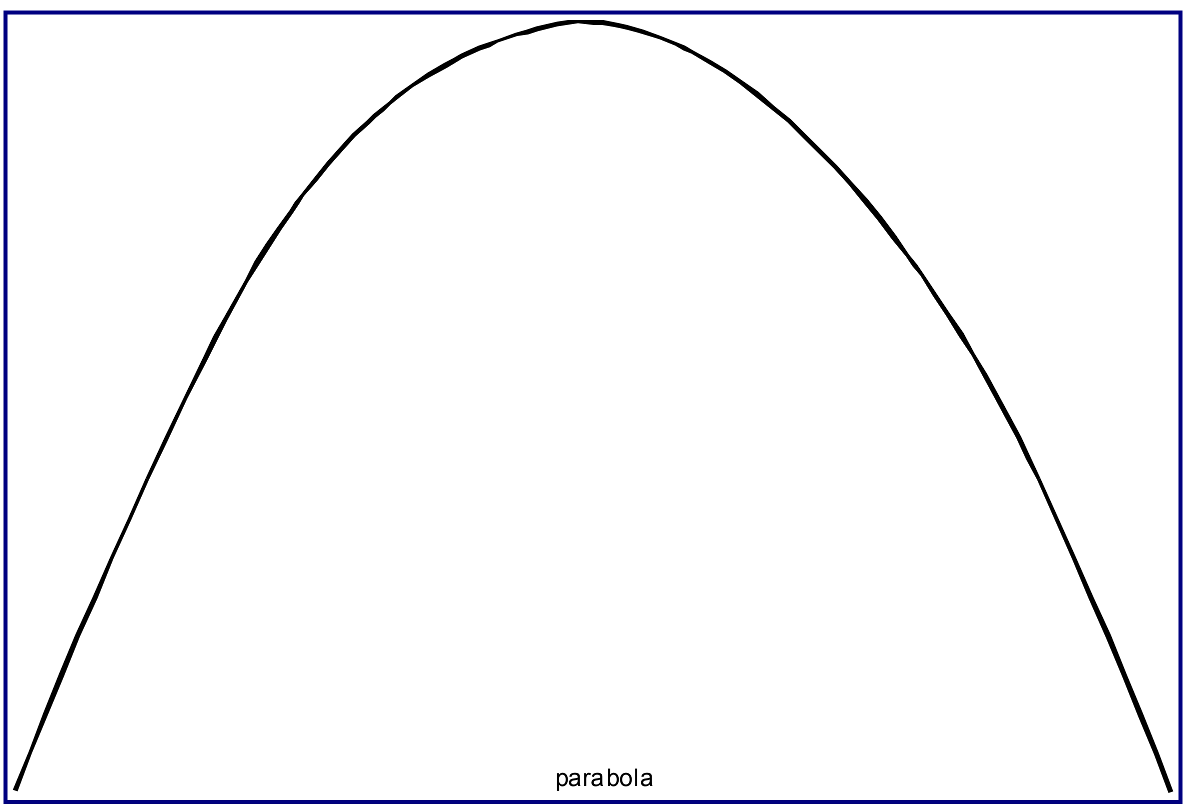
A student or the teacher throws a small ball and matches its flight to a projection of a parabola produced on a screen.

You will need

- ✓ overhead projector transparency of a parabola
- ✓ overhead projector
- ✓ screen
- ✓ small ball
- ✓ some practice

What to do

1. Print out this, or another parabola, onto an OHT.



2. Project the image of a parabola onto a screen and throw the ball so that its shadow follows the same path.

For success, lob the ball as if trying to land it on an imaginary 'shelf' at the top of the parabola.

Practical advice

This demonstration is an eye-catching way to show that the path of a thrown ball really is parabolic. Note that the course does not require students to derive or follow the derivation of a formula for the motion which can be seen to be of the same type as the formula for a parabola.

A good tip to throw the ball successfully is to imagine throwing it to land on a shelf placed at the top of the parabola.

External references

This activity is taken from Advancing Physics Chapter 9, 130D

TAP 207- 2: Diluted gravity - projectile paths

An extension of the diluted gravity experiment is to investigate a diluted projectile path. Get a drawing board and fix a large sheet of paper to it. On top of this fix a piece of carbon paper - face downwards. Tilt the board and then roll a heavy ball bearing across the top of the paper in a horizontal direction.

The path of the ball bearing will be produced on the paper. Different angles of tilt and different path directions can be used. This would be suitable for an introduction to projectiles or at a more advanced level where calculation of the parameters of the paths can be performed.

Apparatus required:

- ✓ Drawing board
- ✓ Large ball bearing
- ✓ Carbon paper
- ✓ White paper

External references

This activity is taken from Resourceful Physics http://resourcefulphysics.org/

TAP 207- 3: Mid-air collisions

Do fired and directly falling objects have the same vertical motion?

Firing something sideways and dropping something at the same time can result in a mid-air collision.

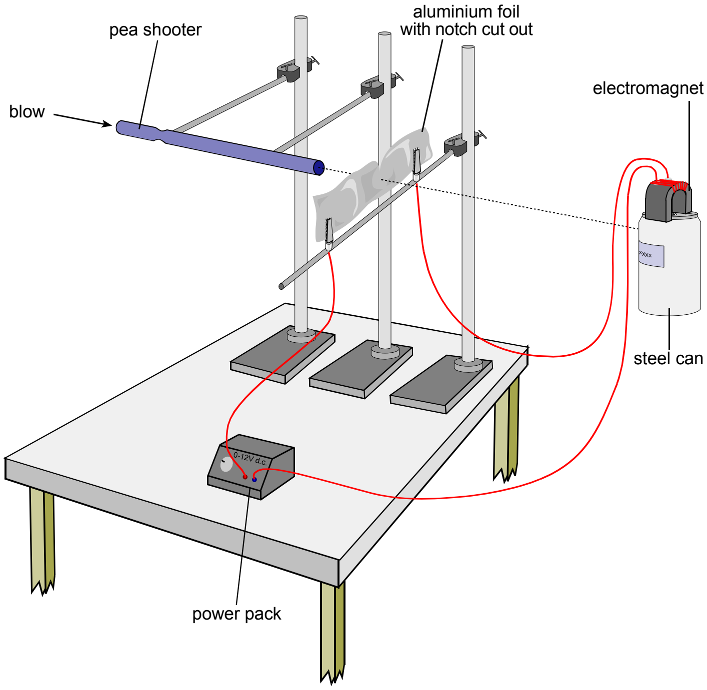
This is often referred to as the Monkey and Hunter experiment, where you can visualise the 'smart' monkey letting go of the branch as the hunter pulls his gun and then being upset when he notes the relative velocity vector is always pointing straight towards him. You may well think of a more humane title.

You will need

- ✓ electromagnet
- ✓ iron can
- ✓ power supply, 12 V
- ✓ aluminium foil with deep notch cut out
- ✓ blowpipe tube
- ✓ ball bearing
- ✓ pair of crocodile clips mounted in a holder
- ✓ 4 mm leads

Remarkable! Two fall vertically the same way

There are a number of ways of performing this demonstration. The essential idea is that both the fired object and the dropped object should start their journeys together.



You may well see the two starting their fall together; you are more likely to hear the collision!

Hearing and seeing

1. Relative motion can be lethal when it causes an unwanted collision. Watch the relative velocity vector with care.

2. Vertical acceleration is quite independent of horizontal movement.

Practical advice

All students should see, and hear, this demonstration at some time in their lives. Now it injects a little life into what might otherwise be a rather dry topic.

Technician's note:

This experiment requires care when setting up. The collision needs to happen before either bullet or monkey hit the floor, for example. That both start falling at the same time is more likely if the smallest possible current is used to activate the electromagnet. A little care and knowledge also helps in designing the circuit breaker. The ball bearing must break the electrical circuit by tearing the foil. It is necessary to make a slit in the foil to initiate the tear. The crocodile clips must be on a non-conducting support.

It is probably best to ensure that the barrel of the blowpipe is horizontal, as this simplifies the discussion.

Alternative approaches

Videos of this event tend to be unconvincing as most equipment does not have a sufficiently high frame rate.

External references

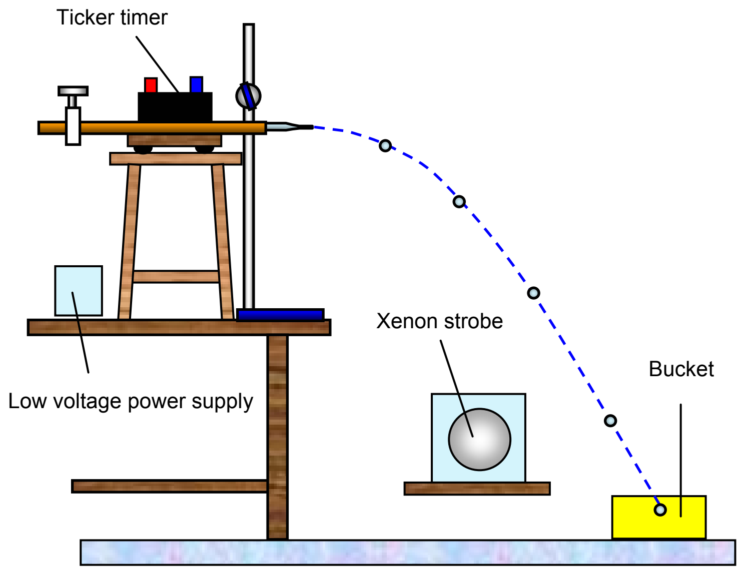
This activity is taken from Advancing Physics Chapter 9, 170D

TAP 207- 4: Pearls in air

This is a classic demonstration designed to show the parabolic path of projectiles in a gravitational field. A water jet is formed by a using the glass part of a dropping pipette fixed to a thin walled rubber tube and connected to the water tap. The rubber tube is passed through an old style ticker timer or over a vibration generator so that the tube is alternately squeezed and released when the device is switched on.

The water jet falls in a parabola from an initial horizontal direction but is also interrupted by the pulsing so that droplets of water are formed instead of a continuous stream. If the arrangement is illuminated with a stroboscope pearl like droplets of water can be made to stand still or move slowly through the air. The constant horizontal velocity and the increasing vertical velocity can be seen by observing the positions of successive drops. To get a permanent record you could make the position of the shadows of the water drops on a screen behind the jet or even photograph it. A truly beautiful demonstration.

An extension of the basic version is the Double Pearls in Air. In this experiment two jets are used from different water taps but with tubes running under the same ticker timer bar. One is adjusted to give a parabola while water simply dribbles out from the other, falling vertically. The vertical acceleration of the drops can then be compared. Of course you can make two parabolas and compare these.



Be safe	Warn pupils about the hazards of strobe lighting and epilepsy.

Theory

Since $h = 1/2gt^2$ and $s = vt$ the equation for the parabolic path for the water is $h = gs^2/2v^2$ where s is the horizontal distance travelled, h the vertical distance and v the horizontal velocity of the jet

Apparatus required

- ✓ Ticker timer plus power supply
- ✓ Two water jets
- ✓ Constant head apparatus, (a large tin with bung and tube will do. See also http://www.practicalphysics.org/go/Apparatus_1027.html for more information.)
- ✓ Bucket
- ✓ Stroboscope

External references

This activity is taken from Resourceful Physics http://resourcefulphysics.org/

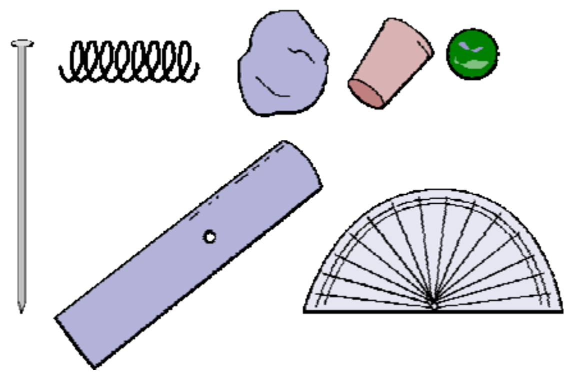
TAP 207- 5: Build and test a marble launcher

Parabolas, projectiles and gravity

Start with the construction of a simple marble launcher. Then analyse its performance via a series of investigations. Building confidence in the use of the kinematic equations is the main objective, although many measurement and design skills can be developed.

You will need

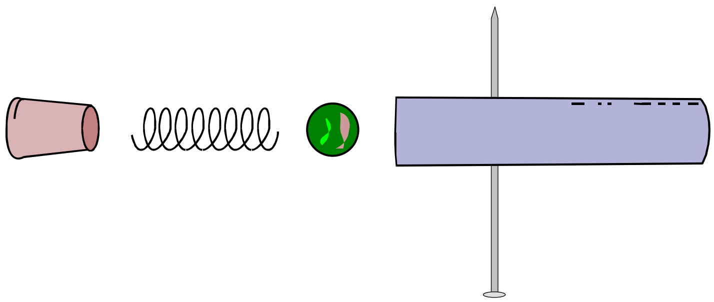
- ✓ a protractor
- ✓ a metre rule
- ✓ a small sand pit
- ✓ a means of measuring speed
- ✓ safety spectacles
- ✓ a compression spring
- ✓ 1 cm diameter plastic conduit with rubber bung to fit
- ✓ marble and / or ball bearing to fit tube
- ✓ drill and large nail

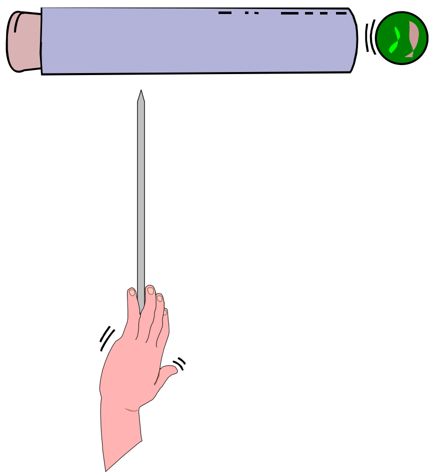


Wear safety spectacles

The projectiles are likely to be smaller than the eye socket, and may not always be very well aimed!

Building and evaluating the performance of a launcher





You may be given the launcher to assemble or be asked to construct one from drawings. Removing the nail smartly will result in a clean launch. You will want to measure the angle of launch and the range and maximum height of the flight.

1. Design and carry out an experiment to measure the exit speed of the marble for a given spring compression setting.

2. Now use the kinematic equations for a given angle of launch, ignoring air resistance, to see if you can land the marble in the sand pit. Is it reasonable to ignore air resistance here? Is the marble flight adequately described by the kinematic equations, used in two dimensions?

All very predictable?

1. The equations allow you to predict, perhaps surprisingly well, where the marble will land.

2. At the end of this activity you should be confident in using the equations.

3. There are some limitations to be careful of when applying the equations to a real projectile launcher.

Practical advice

The launcher activity will allow your students to get some practical work done while studying motion in a gravitational field. You may want them to build the launcher themselves or provide a kit for assembly.

The launcher design described in this activity was based on the beautiful but expensive Pasco device. Students like to use the equations to get the marble into the sand-pit. Competition soon sets in.

In trying to measure the muzzle velocity some students might try an indirect approach and you might revise some energy ideas in the first part of the activity, although this is not strictly necessary. Most students will be happy with simple conservation arguments from pre-16 course.

Alternative approaches

A more controlled version might be to organise an activity around one pre-made launcher.

Be safe

Students should wear eye protection, at least bearing the 'F' impact code (although 'B' would be better)' during this activity. The projectiles are small and may well transfer significant amounts of kinetic energy on impact. For the same reasons you may want to limit the materials and construction techniques used. Considering the disposition of the firing ranges before live firing commences may also limit the collateral damage to fixtures and fittings.

	**Wear safety spectacles** The projectiles are likely to be smaller than the eye socket, and may not always be very well aimed!

External references

This activity is taken from Advancing Physics Chapter 9, 172E

TAP 207- 6: Challenges in target archery

Here you can work on predicting the positions of objects that are accelerated. One archery competition requires archers to fire a total of 90 arrows for a maximum possible score of 900. The targets are 1.22 m in diameter at 60, 50 and 40 metres distance. You can model the motion of the arrow to find out what problems the archer faces.

The arrow leaves the bow at 60 m s^{-1} and travels at almost this speed horizontally for the whole of its flight.

The arrow, of course, falls because of the acceleration due to gravity. You can find its position at any moment by working out how far it has moved horizontally and how far it has fallen vertically.

1. The archer shoots the arrow horizontally at the 40 m target. How far does it drop over this range?

2. How would the archer make allowance for this fall? Think carefully before committing yourself.

3. Now try to calculate the fall at 50 m and 60 m.

4. 50 years ago the release speed of an arrow was about 30 m s^{-1}. What effect would this have on the vertical distance the arrow fell? Calculate the drop for a range of 60 m to check your answer.

5. We have ignored the effect of air resistance in these calculations. How could you take account of it? You can explore it most easily by making a computer model of the motion.

Practical advice

An example where vectors and simple kinematics give insight into a phenomenon.

Social and human context

Accurate archery was important in many spheres – often not in warfare.

Answers and worked solutions

1. Time of flight

$$= \frac{40\,\text{m}}{60\,\text{m}\,\text{s}^{-1}} = 0.67\,\text{s}$$

Vertical drop

$$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81\,\text{m}\,\text{s}^{-2} \times \left(\frac{2}{3}\,\text{s}\right)^2 = 2.2\,\text{m}$$

2. Aim above the target.

3. Similar calculations

$$h_{50} = \frac{1}{2} \times 9.81\,\mathrm{m\,s^{-2}} \times \left(\frac{5}{6}\,\mathrm{s}\right)^2 = 3.4\,\mathrm{m}$$

$$h_{60} = \frac{1}{2} \times 9.81\,\mathrm{m\,s^{-2}} \times (1\,\mathrm{s})^2 = 4.9\,\mathrm{m}$$

4. Twice the trip time so four times fall, at 60 m now drops 20 m.

External references

This activity is taken from Advancing Physics Chapter 9, 110s