TAP 404-1: GPE in a constant field

Discussion:

Potential energy in a non-uniform field

Now you can extend these ideas to potential energy in a non-uniform field.

All the previous examples involved potential energy changes near the surface of the Earth. What would be the problem if we wanted to use the same equation to work out energy changes for, say, a rocket launched to the Moon? (The gravitational field strength is not constant – the value of g changes.)

We therefore need another way of calculating GPE changes in non-uniform fields. The full treatment of how we arrive at this formula requires off-syllabus calculus that would actually be accessible to more able students. We find that we can calculate GPE of a mass m at a point distance r from a (point or spherical) mass M by:

GPE = –GMm/r

There are several very important points to note about this equation:

1. We know that the further you get from an object, the higher your GPE relative to it. (As something must have done more work against gravity to get you there). Thus when you are infinitely far away, you have as high a GPE relative to it as possible. We choose (arbitrarily) to make the value of GPE of all bodies at infinity zero. Then since this is the highest value of GPE, all real values of GPE (closer than infinity) must be negative. Therefore the minus sign in the equation is NOT optional; it must always be included and all values of potential energy in a gravitational field are negative. (This is not the case when we come on to electric fields, because they can be repulsive too).

2. Note that we have written GPE here, and not “Change in GPE”. By defining a point relative to which all GPE is measured, we can now talk about absolute values of GPE rather than just changes. This point is at infinity (see note 1 above).

3. Note that GPE follows an inverse proportion law (1/r) and not an inverse square law ($1/r^2$)

Discussion:

Potential

The weight of an object in the Earth’s gravitational field depends upon the mass of the object (as well as the mass of the Earth). However, as we have already seen, the field strength at a point is independent of the object placed there (because it is defined as force per unit mass of the object). Thus we can think of field strength as a property of the field at a point, and not the particular object placed there.

Similarly, the GPE of a body at a place in the Earth’s field depends upon the mass of the object (as well as the mass of the Earth). How do you think we can get a quantity related to energy in the field at a point, which does not depend upon the object placed there? (By looking at the GPE per unit mass of the object, thus removing the dependence on the mass of the object just as we did with field strength.)

We define the potential at a point in a field as the gravitational potential energy per unit mass placed at that point in the field. We can get equations for potential using this definition. For a field due to a (point or spherical) mass M, we have:

GPE = –GMm/r

And so the potential, V, is given by:

V = (–GMm/r) / m = –GM/r

A few points to make:

1. This only relates to the field due to a (point or spherical) mass M.

2. V is measured in J kg^{-1}. It follows an inverse proportion law (1/r) not inverse square ($1/r^2$).

3. Just as with the equation for GPE, the minus sign is not optional. All real potentials are negative, and the zero of potential is at infinity (since all objects have zero GPE at infinity).

4. Potential, like field strength is a property of the field at a point, and is independent of the object placed there. Two objects with different masses at the same point in the field are subject to the same potential, but have different potential energies.

5. For uniform fields (e.g. close to the surface of the Earth), we can use

change in potential = gh

(since change in GPE = mgh and potential = GPE/m).

[Potential will be returned to again when we study electric fields. There, differences in potential (or potential differences, pds) are what we often call voltages.]

Worked examples:

Potential energy and potential