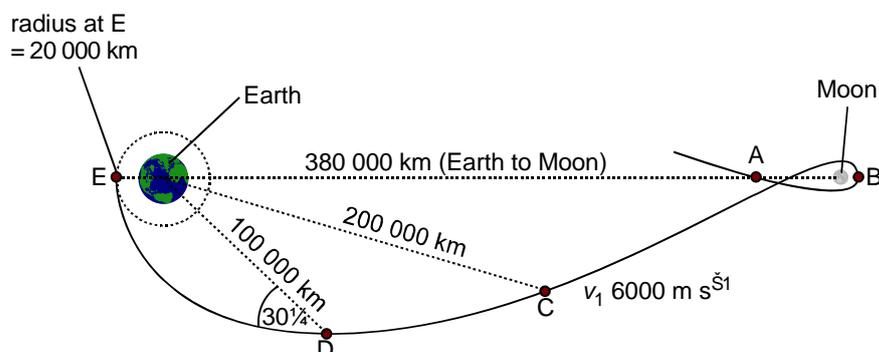# TAP 404-3 Gravitational field between the Earth and the Moon

The dynamics of space travel

These questions pose a number of short problems about the dynamics of the motion of a spacecraft, returning from the Moon to the Earth. The mass of the spacecraft is 2000 kg.

radius at E
= 20 000 km

Earth

Moon

E

380 000 km (Earth to Moon)

A

B

200 000 km

100 000 km

30¼·

D

C

v_1 6000 m s⁻¹

The diagram shows its path and is drawn to scale (except for the diameters of the Earth and the Moon which are shown larger than they should be). Distances given are to the centre of the Earth.

mean radius of Earth as 6.4×10^6 m, G x M for the Earth = 4.0×10^{14} N m² kg⁻¹

Gravitational force and potential energy

1.   At point *A* the spacecraft has zero acceleration when its thrust motor is switched off. Suggest a reason why.

2.   At point B, again without any motor thrust, the speed of the spacecraft is constant, though it does have acceleration. (At B, the craft is travelling at right angles to the line joining it to the Moon's centre.) Explain this observation.

3.   At C, the velocity is 6000 m s⁻¹, as shown. It is required to alter course by an angle of 1/100 rad, without changing speed.

In what direction must the thrust be?

Calculate the length of time for which the motor thrust of 12 000 N be maintained?

4.   In which direction is the spacecraft accelerating at D if the thrust motor is not in use?

        Calculate the magnitude of the change its in momentum of the space craft over a time of 100 s. Ignore any effect of the Moon in this question.

5.   Make a rough calculation to show it is fair to ignore the effect of the Moon in question 4.

        Ignore any effects of the Moon in questions 6–8.

6.   The thrust motors are **not** used between C and E.

        Calculate the change in gravitational potential energy of the spacecraft change between C and E?

        Explain whether it is an increase or a decrease, and whether the answer would have been different if the motors had been used.

7.   At *C* the spacecraft's velocity is 6000 m s^{-1}.

        Use your answer to calculate the kinetic energy of the spacecraft at E.

8   What kinetic energy would the spacecraft have at E if, instead of having come in from Moon orbit, the spacecraft were in permanent orbit around the Earth through E?

Hints

1.   Consider both the magnitude and direction of the forces acting on the spacecraft at A.

2.   Both the Moon and the spacecraft will have mass and there will be a gravitational force of attraction between them.

3.   Remember acceleration can be produced by a change in direction as well as speed.

5.   Remember this is a scale drawing.

6.   The gravitational potential is 0 J kg^{-1} at infinity.

7.   You can assume conservation of mechanical energy when there is no work done against friction to be considered and the motors are not in use.

Practical advice

It is helpful to remind students that the distances in the drawings are to scale and they will need to measure them with a ruler (in question 5).

Social and human context

These are useful questions to consider when discussing manned missions to the Moon. There are many video clips that could be useful to put these journeys into a historical perspective. This could lead to discussions on the reasons for the 'space race'; consideration as to which explorations are seen as prestigious in the present day; and the social and economic implications of the high budget demands of pure and applied research. Is such research likely to help or hinder increased international cooperation?

Answers and worked solutions

1. The force of attraction between the spacecraft and Earth is equal in magnitude but opposite in direction to the force of attraction between the spacecraft and the Moon. There is no net force, so no acceleration.

2. The gravitational force is at right angles to the direction of motion, so there is no change in speed. The direction, however, will change so that the force is always acting from the centre of the spacecraft to the Moon. Since there is a change in direction, there is a change in velocity, therefore there is acceleration.

3. The thrust acts at right angles to the speed, since this does not change.
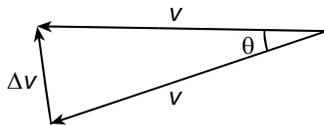
   As shown in the vector triangle the change in velocity Δv is $v\theta$ (for a small angle θ), F = $(m\Delta v)/t$

   $$t = \frac{m\Delta v}{F} = \frac{mv\theta}{F} = \frac{2000 \text{ kg} \times 6000 \text{ ms}^{-1} \times 1/100 \text{ rad}}{12\,000 \text{ N}} = 10 \text{ s.}$$

4. If the thrust motors are not used, the only force acting on the spacecraft is the gravitational force of attraction between the Earth and the spacecraft; this acts towards the centre of the Earth.

   F t = change in momentum and since

   F = $-$ (G M m) / R^2

   $$Ft = \frac{(-4 \times 10^{14} \text{ Nkgm}^{-2}) \times 2000 \text{ kg}}{(6.4 \times 10^{6} \text{ m})^2} \times 100 \text{ s} = 800 \text{ Ns.}$$

5. The measured distance from D to the Moon is three times the distance from D to the Earth. This means that the gravitational force of attraction on the spacecraft due to the Moon (an inverse square law) would be 1/9 that due to the Earth **if** the Earth and Moon had the same mass. But the Moon's mass is smaller than Earth's, and thus it is fair to ignore it in these calculations.

6. The change in gravitational potential energy =

$$-\left(\frac{GMm}{R_2} - \frac{GMm}{R_1}\right) = GMm\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$(4 \times 10^{14} \ \text{Nm}^2 \ \text{kg}^{-1}) \times 2000 \ \text{kg}\left(\frac{1}{2 \times 10^8 \ \text{m}} - \frac{1}{2 \times 10^7 \ \text{m}}\right) = -3.6 \times 10^{10} \ \text{J}.$$

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Since gravitational potential is 0 at infinity and work needs to be done to reach infinity, the potential energy is less at a distance closer to the centre of a gravitational field, in this case at E. Since potential energy is a scalar quantity and does not depend on the path taken, this value is the same as if the motors had been used to move between C and E.

7. From conservation of mechanical energy (no work done against friction, motors not used), the decrease in potential energy = increase in kinetic energy, 3.6×10^{10} J. At C, the kinetic energy was already $0.5 \times 2000 \ \text{kg} \times (6000 \ \text{m s}^{-1})^2 = 3.6 \times 10^{10}$ J so the total kinetic energy is 7.2×10^{10} J.

8. For a circular orbit:

$$F = \frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\text{so } v^2 = \frac{GM}{R}$$

$$E_K = \tfrac{1}{2}mv^2 = \frac{mGM}{2R} = \frac{2000 \ \text{kg} \times (4.0 \times 10^{14} \ \text{Nm}^2 \ \text{kg}^{-1})}{2 \times (2 \times 10^7 \ \text{m})} = 2 \times 10^{10} \ \text{J}$$

External reference

This activity is taken from Advancing Physics chapter 11, 120D