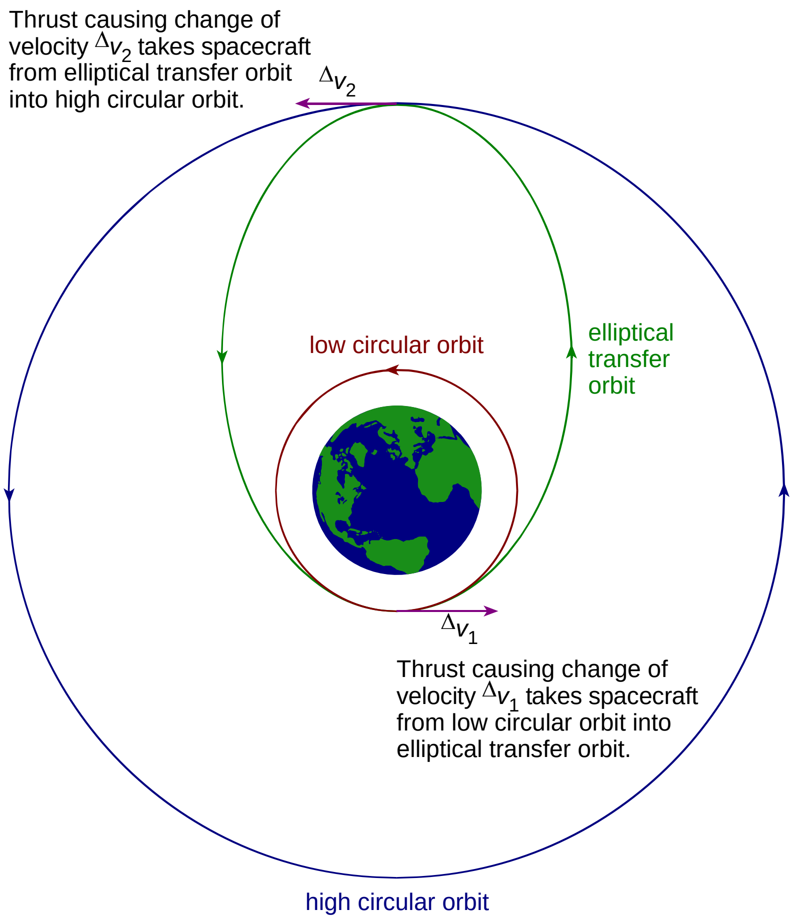
TAP 403-3: Changing orbits

Launching a satellite into a circular geostationary orbit is a two-stage operation, involving a lower circular orbit – called a parking orbit – and an elliptical transfer orbit, which is sometimes called a Hohmann orbit.



A satellite is launched into geostationary orbit (radius 42 200 km) via a parking orbit at a height 300 km above Earth's surface. Increases in speed to take the satellite into and out of the elliptical transfer orbit can be regarded as instantaneous.

radius of Earth to be 6370 km.

A few questions

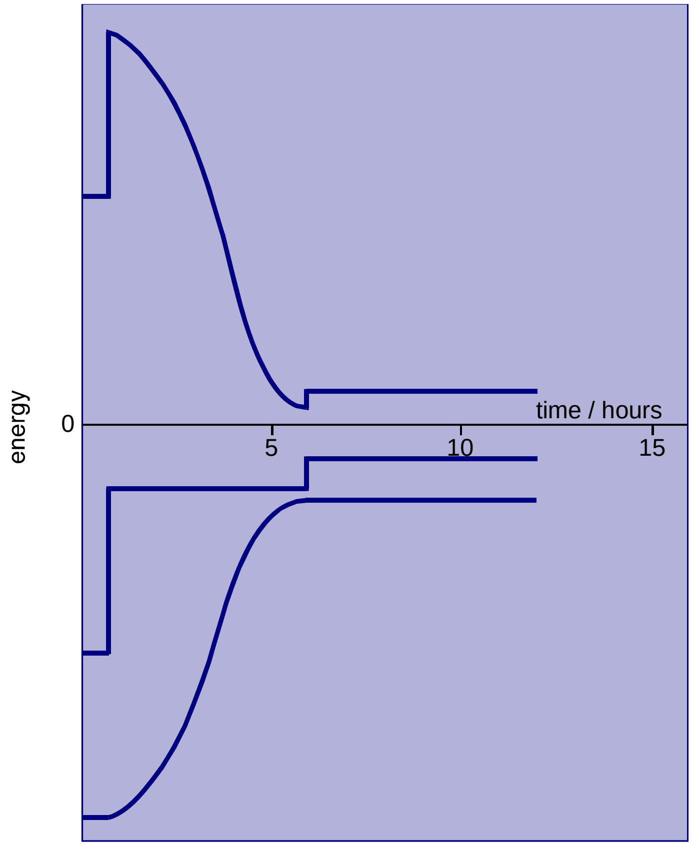
1. Calculate the speed of the satellite in the geostationary orbit.

2. Use Kepler's third law ($\text{period}^2 \propto \text{mean orbit radius}^3$) to calculate the period of the satellite in the parking orbit. Use the period to calculate the speed of the satellite in the parking orbit.

3. Is the kinetic energy of the satellite in the geostationary orbit greater or less than when it is in the parking orbit?

4. Find the average distance for the transfer orbit, by averaging largest and smallest distances. Hence, calculate the time taken for the satellite to travel from parking orbit to geostationary orbit.

5. The graphs show the energy of the satellite before, during and after the transfer between parking and geostationary orbit. Identify which line refers to kinetic energy, which to potential energy and which to total energy.



6. Sketch the path of the satellite around Earth for the 12 h shown.

Practical advice

This set of questions tries to draw together what students know of elliptical and circular orbits, emphasising the energies needed. Students may need a verbal briefing on what is required for question 6.

Answers and worked solutions

1.

$$v = \frac{s}{t} = \frac{2\pi \times 42\,200\ \text{km}}{24 \times 60 \times 60\ \text{s}} = 3.07\ \text{km s}^{-1}.$$

2.

$$\frac{T^2}{(1\ \text{day})^2} = \left(\frac{6370\ \text{km} + 300\ \text{km}}{44\,200\ \text{km}}\right)^3$$

$$T = \sqrt{\left(\frac{6670}{42\,200}\right)^3 \times (1\ \text{day})^2} = 0.0628\ \text{days} = 90.5\ \text{minutes}$$

so

$$v = \frac{s}{t} = \frac{2\pi \times 6670\ \text{km}}{90.5 \times 60\ \text{s}} = 7.72\ \text{km s}^{-1}.$$

3. E_k depends on v^2 so the kinetic energy is less in the geostationary orbit.

4.

$$R = \frac{6670\ \text{km} + 42\,200\ \text{km}}{2} = 2.44 \times 10^4\ \text{km}$$

and the time for half an orbit will be

$$\frac{T^2}{(1\text{day})^2} = \left(\frac{2.44 \times 10^4\ \text{km}}{44200\ \text{km}}\right)^3$$

$$T = \sqrt{\left(\frac{2.44 \times 10^4\ \text{km}}{44200\ \text{km}}\right)^3 \times (1\text{day})^2} = 0.44\ \text{day}$$

$$\frac{T}{2} = 0.22\ \text{day} = 5.3\ \text{hr}$$

5. Upper line = kinetic energy; middle line = total energy; lower line = gravitational potential energy.

6. The diagram should reflect the times read from the graph 0.7 h in the parking orbit (half an orbit); 5.3 h in elliptical transfer orbit (half ellipse); 6 h in geostationary orbit (1/4 of orbit). Total 1/2 + 1/2 + 1/4 = 1 1/4 orbits.

External reference

This activity is taken from Advancing Physics chapter 11, 230D