TAP 403-2: Using Kepler's third law

Take radius of Earth to be 6 370 km.

1. The radius of a geostationary orbit is 42 200 km. Use this fact together with the constancy of R^3 / T^2 to estimate the height above the Earth's surface of a satellite whose circular orbit is completed in 90 minutes. How many times a day would such a satellite orbit the Earth?

2. Low-orbiting Earth satellites usually have orbital periods in the range 90 to 105 minutes. What range of heights does this correspond to?

3. 90 minutes is a typical orbital period for a military reconnaissance satellite, and 100 minutes for a civilian Earth observation satellite. Can you suggest a reason for this difference?

4. Kepler's laws were formulated for *elliptical* orbits (of which the circular orbit is a simple special case). The '*R*' of the third law is the semi-major axis (found as the average of the maximum and minimum distances between a satellite and the body it orbits). You can see how this works by looking at data for Sputnik 1, the first artificial satellite, which was launched on 4 October 1957 and, slowly losing energy due to the effects of atmospheric friction, spiralled back to Earth 3 months later. Complete the following table of data:

	4 October 1957	**25 October 1957**	**25 December 1957**
Orbital period / minutes	96.2	95.4	91.0
Minimum height / km	219	216	196
Maximum height / km	941	866	463
Mean height / km			
Mean radius / km			
R^3 / T^2 three significant figures			

Did the orbit become less elliptical as time passed?

Practical advice

If Kepler’s laws are not in the course specification: you may want to use only a few questions from this set.

Answers and worked solutions

1.

$$\frac{42\,200\ \text{km}^3}{24\ \text{h}^2} = \frac{R^3}{1.5\ \text{h}^2}$$

so

$$R = \sqrt[3]{\frac{42\,200\ \text{km}^3 \times 1.5\ \text{h}^2}{24\ \text{h}^2}} = 6650\ \text{km}.$$

Therefore

$$h = 6650\ \text{km} - 6370\ \text{km} = 280\ \text{km}.$$

$$\text{number of orbits} = \frac{24\ \text{hours}}{1.5\ \text{hours}} = 16$$

2.

$$\frac{R_{90}^3}{T_{90}^2} = \frac{R_{105}^3}{T_{105}^2}$$

$$\frac{(6650\ \text{km})^3}{(90\ \text{minutes})^2} = \frac{R_{105}^3}{(105\ \text{minutes})^2}$$

$$R_{105}^3 = \frac{(2.9 \times 10^{11}\ \text{km}^3) \times (1.1 \times 10^4\ \text{minutes}^2)}{8.1 \times 10^3\ \text{minutes}^2}$$

$$R = 7330\ \text{km}$$

$$\text{height} = 7330\ \text{km} - 6370\ \text{km} = 960\ \text{km}$$

range is 280 km to 960 km

3. Low orbits give smaller image detail (is it a battlefield tank?); higher orbits give greater coverage and endurance (because there is less atmospheric friction).

4.

	4 October 1957	**25 October 1957**	**25 December 1957**
Orbital period / minutes	96.2	95.4	91.0
Minimum height / km	219	216	196
Maximum height / km	941	866	463
Mean height / km	580	541	330
Mean radius / km	6950	6911	6700
R^3 / T^2 three significant figures	36×10^6	36×10^6	36×10^6

The Kepler ratio for each case is the same; the deviation from the mean height decreases, so the orbit becomes more like a circle.

Average orbit time was 93.6 minutes. In 3 months (90 days) it made approximately

$$\frac{90\text{ days} \times 24\text{ hours day}^{-1} \times 60\text{ min hr}^{-1}}{94\text{ min}} = 1400\text{ orbits}$$

External reference

This activity is taken from Advancing Physics chapter 11, 10D