TAP 403-1: Worked examples – Orbital Motion

Student sheet

Data required:

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

mass of the Earth = 6.0×10^{24} kg

radius of the Earth = 6.4×10^{6} m

mass of Jupiter = 1.9×10^{27} kg

radius of Jupiter = 7.2×10^{7} m

Jupiter's day length = 10 hours

1) What is the only force acting on a single planet orbiting a star? Write down an expression for this force. If the planet moves in a circular orbit of radius *r*, at constant speed *v*, write down an expression for this speed in terms of the period *T* of the orbit.

Because the orbit is circular, the planet must experience a centripetal force of size mv^2/r. Use this fact and the 2 expressions you have written down to prove Kepler's third law, which states that the square of the time period of the planet's orbit is proportional to the cube of the radius of the orbit.

2) Use Kepler's third law, $T^2 \propto r^3$, to answer this question. Two Earth satellites, A and B, orbit at radii of 7.0×10^{6} m and 2.8×10^{7} m respectively. Which satellite has the longer period of orbit? What is the ratio of orbital radii for the two satellites? What, therefore, is the ratio of the cubes of the orbital radii? What, therefore, is the ratio of the squares of the orbital periods? Finally therefore, what is the ratio of the satellites' orbital periods?

3) What is a geostationary satellite? Describe and explain the orbit of such a satellite. What might such a satellite be used for? With the help of your final expression in question 1, work out the orbital radius of such a satellite. What height is this above the Earth's surface?

4) Suppose we wanted to place a satellite in "jovi-stationary" orbit around Jupiter (the same as geostationary, but around Jupiter, not Earth). What orbital period would it need? What orbital radius would this correspond to?

Orbital Motion – Teacher's Sheet

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$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

mass of the Earth = 6.0×10^{24} kg

radius of the Earth = 6.4×10^{6} m

mass of Jupiter = 1.9×10^{27} kg

radius of Jupiter = 7.2×10^{7} m

Jupiter's day length = 10 hours

1) What is the only force acting on a single planet orbiting a star? Write down an expression for this force. If the planet moves in a circular orbit or radius *r*, at constant speed *v*, write down an expression for this speed in terms of the period *T* of the orbit.

Because the orbit is circular, the planet must experience a centripetal force of size mv^2/r. Use this fact and the 2 expressions you have written down to prove Kepler's third law, which states that the square of the time period of the planet's orbit is proportional to the cube of the radius of the orbit.

Gravitational attraction GMm/r^2.

$v = 2\pi r/T$

GMm/r^2	=	mv^2/r
GM/r	=	$m\,(2\pi r/T)^2$
GM/r	=	$4\pi^2 r^2 m/T^2$
T^2	=	$(4\pi^2/GM)\, r^3$

2) Use Kepler's third law, $T^2 \propto r^3$, to answer this question. Two Earth satellites, A and B, orbit at radii of 7.0×10^6 m and 2.8×10^7 m respectively. Which satellite has the longer period of orbit? What is the ratio of orbital radii for the two satellites? What, therefore, is the ratio of the cubes of the orbital radii? What, therefore, is the ratio of the squares of the orbital periods? Finally therefore, what is the ratio of the satellites' orbital periods?

By Kepler's third law, orbital period increases with orbital radius. Thus B has the longer orbital period.

$r_B/r_A = 2.8 \times 10^7 / 7.0 \times 10^6 = 4$

$(r_B/r_A)^3 = 4^3 = 64$

By Kepler's third law, $(T_B/T_A)^2 = (r_B/r_A)^3 = 64$

Thus T_B/T_A = square root of 64 = 8

3) What is a geostationary satellite? Describe and explain the orbit of such a satellite. What might such a satellite be used for? With the help of your final expression in question 1, work out the orbital radius of such a satellite. What height is this above the Earth's surface?

A satellite that appears to be stationary over a point on the Earth's equator. The orbit of such a satellite is circular and over the Earth's equator as the satellite's orbital centre is the centre of the Earth, and the only points on Earth that orbit its centre are those on the equator.

Such a satellite might be used for communications, e.g. satellite broadcasting.

$T^2 = (4\pi^2/GM)\ r^3$

$r^3 = (GM/4\pi^2)\ T^2$

$= (6.67 \times 10^{-11} \times 6.0 \times 10^{24} / 4\pi^2) \times (24 \times 60 \times 60)^2$

$= 7.567 \times 10^{22}\ m^3$ (4sf)

Therefore, $r = 4.23 \times 10^7$ m (3sf)

Height above Earth's surface is $4.23 \times 10^7 - 6.4 \times 10^6 = 3.59 \times 10^7$ m or 36,000 km.

4) Suppose we wanted to place a satellite in "jovi-stationary" orbit around Jupiter (the same as geostationary, but around Jupiter, not Earth). What orbital period would it need? What orbital radius would this correspond to?

10 hours

$r^3 = (GM/4\pi^2)\ T^2$ (see previous question)

$= (6.67 \times 10^{-11} \times 1.9 \times 10^{27} / 4\pi^2) \times (10 \times 60 \times 60)^2$

$= 4.160 \times 10^{24}\ m^3$ (4sf)

Therefore, $r = 1.61 \times 10^8$ m (3sf)