

## TAP 413- 6: Charged particles moving in a magnetic field

### Instructions

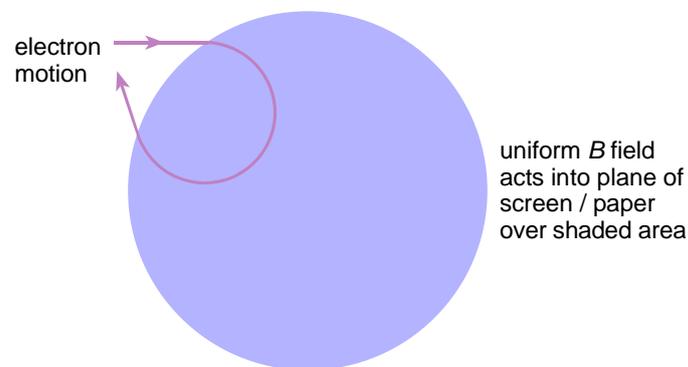
This question set contains two groups of questions. Answer all the questions in the spaces provided. The following data will be needed:

- magnitude of the electronic charge =  $1.6 \times 10^{-19}$  C
- mass of electron =  $9.1 \times 10^{-31}$  kg

**Questions 1– 5** are about the motion of charged particles in a bubble chamber.

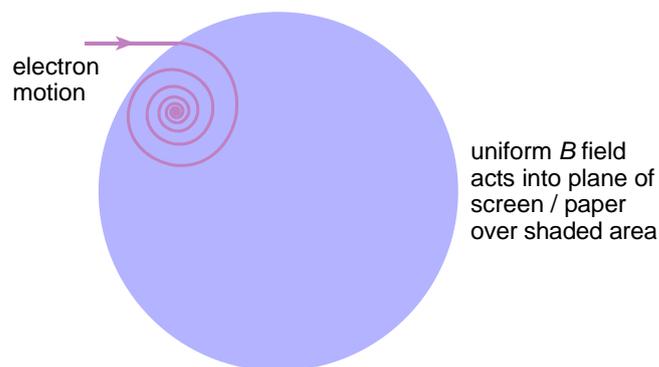
1. An electron gun in a vacuum accelerates electrons up to a kinetic energy of  $2.9 \times 10^{-16}$  J. Show that the speed acquired by each electron is  $2.5 \times 10^7$  m s<sup>-1</sup>.

The electron beam enters a region of uniform magnetic field of strength,  $B$ , perpendicular to the beam. The magnetic field causes the beam to follow a circular path as in the diagram below.

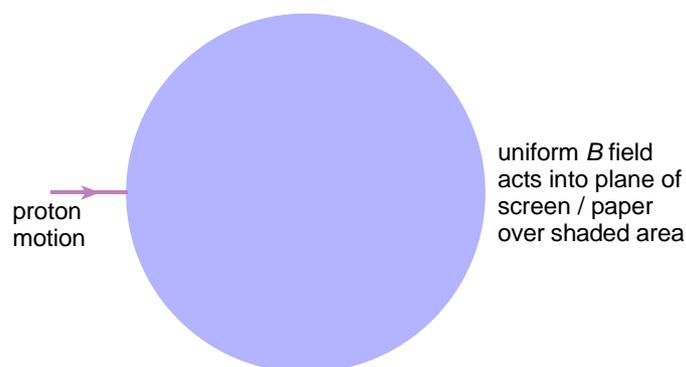


2. Show that the force experienced by the electron is about  $8.8 \times 10^{-15}$  N, when  $B$  is  $2.2 \times 10^{-3}$  T.
3. Use your answer to question 2 to find the radius of this circular path.

4. Evidence for the motion of electrons in magnetic fields can be observed from the trail of bubbles they leave as they pass through liquid hydrogen. In these bubble-chamber experiments a single electron tends to produce a track which is a spiral rather than a circle. Explain why.



5. On the diagram below draw the likely path of a proton travelling at the same speed in an identical field.



**Questions 6–10** are about the motion of charged particles in magnetic fields and the cyclotron frequency.

6. Explain why a charged particle, moving with a constant speed  $v$  perpendicular to a uniform magnetic field  $B$ , will follow a circular path.

7. Show that for a particle of mass  $m$  and charge  $q$  the radius of the circular path is given by the expression  $r = mv/Bq$ .
  
8. Using your answer to question 7, show that the frequency of this circular motion, known as the cyclotron frequency, is given by the expression  $f = qB/2\pi m$ .
  
9. Some astrophysicists believe that the radio signals of  $10^9$  Hz reaching us from Jupiter are emitted by electrons orbiting in Jupiter's magnetic field. Assuming the frequency of the radio emission is identical to the cyclotron frequency; find the strength of the magnetic field around Jupiter.
  
10. The electrons lose energy as they emit radiation. What effect, if any, will this have on the frequency of the radio signals detected? Explain your answer.

### Hints

5. Think about the charge and mass of the proton.
  
10. What happens to the speed of the proton as it loses energy?

## Practical advice

Both sets of questions link with work in particle physics. The radio emissions from electrons orbiting in the magnetic field of Jupiter are explained using the idea of cyclotron frequency.

## Answers and worked solutions

1.

$$E_k = \frac{1}{2}mv^2$$

so

$$v = \sqrt{\frac{2 \times (2.9 \times 10^{-16} \text{ J})}{(9.1 \times 10^{-31} \text{ kg})}} = 2.5 \times 10^7 \text{ m s}^{-1}.$$

The kinetic energy is considerably less than the electron's rest energy, so the Newtonian approximation can be used.

2.

$$\begin{aligned} F &= Bqv \\ &= (2.2 \times 10^{-3} \text{ T}) \times (1.6 \times 10^{-19} \text{ C}) \times (2.5 \times 10^7 \text{ m s}^{-1}) \\ &= 8.8 \times 10^{-15} \text{ N.} \end{aligned}$$

3.

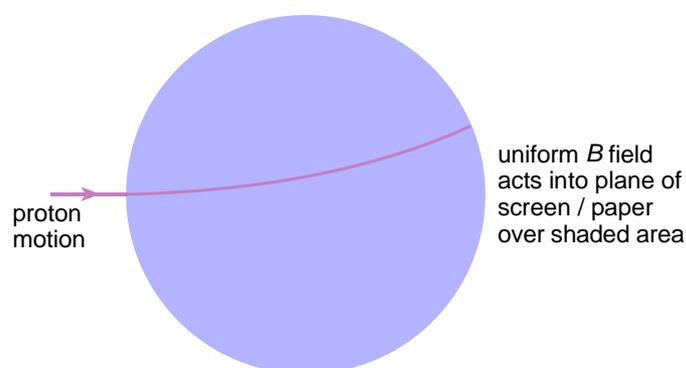
$$F = mv^2 / r$$

so

$$r = \frac{(9.1 \times 10^{-31} \text{ kg}) \times (2.5 \times 10^7 \text{ m s}^{-1})^2}{8.8 \times 10^{-15} \text{ N}} = 6.5 \times 10^{-2} \text{ m.}$$

4. Electrons lose speed through collisions with other particles. Since  $r = mv/Be$ ,  $r$  decreases as  $v$  decreases.

5.



6. The force on the particle is always perpendicular to  $v$ .

7. Equate  $qvB$  to  $mv^2/r$

8. Frequency  $f = v / 2\pi r$ . Substituting  $r$  from question 7 gives  $f = qB/2\pi m$ .

9.

$$B = \frac{2\pi fm}{q} = \frac{2\pi \times 10^9 \text{ Hz} \times (9.1 \times 10^{-31} \text{ kg})}{1.6 \times 10^{-19} \text{ C}}$$
$$= 3.6 \times 10^{-2} \text{ T.}$$

10. There will be no effect since the cyclotron frequency depends only upon  $B$ ,  $q$  and  $m$  that do not change under normal conditions.

### External reference

This activity is taken from Advancing Physics chapter 16, 150S