TAP 603- 4: Speed of sound and speed of molecules

Calculating and comparing speeds

Look at the values of the speed of sound in a gas and the speed of its molecules. You will find that they are comparable in size, with the speed of the molecules always a bit greater, and you can think about why this should be true.

Comparing speeds of sound

Here is a table of measured values of the speed of sound in three gases:

Gas	Molar mass / g	Speed of sound at 273 K / m s^{-1}	Speed of sound at 300 K / m s^{-1}
Helium	4	972.5	1019
Nitrogen	28	337.0	355.5
Carbon dioxide	44	257.4	269.8

1. Which gas has the highest speed of sound, at either temperature?

2. Which gas has the least massive molecules?

3. Which gas has the lowest speed of sound, at either temperature?

4. Which gas has the most massive molecules?

5. At which temperature is the speed of sound the higher?

6. At which temperature are the molecules moving faster?

Comparing speeds of molecules

The next questions may suggest to you a reason for the pattern you have seen.

7. If there are *N* molecules in an ideal gas at temperature T, pressure P, volume V then

$$PV = NkT = \tfrac{1}{3} Nm\overline{v^2},$$

where the molecules have mass m and mean square speed $\overline{v^2}$, and k is the Boltzmann constant.

Show that the mean square speed is given by

$$\overline{v^2} = \frac{3kT}{m}.$$

8. Calculate the mass of a helium atom, given that 4 g (= 0.004 kg) of helium contains N = 6.02×10^{23} atoms.

9. Calculate the square root of the mean square speed for helium atoms, at 300 K, given that the Boltzmann constant k = 1.38×10^{-23} JK^{-1}.

10. The masses of nitrogen molecules and helium atoms are in the ratio 28 / 4. What should be the ratio of their mean square speeds at any given temperature?

11. Using the answer to question 9, predict the square root of the mean square speed (the rms speed) for nitrogen molecules at 300 K.

12. Repeat question 9 for carbon dioxide molecules.

Comparing speeds of sound and speeds of molecules

A sound wave in a gas consists of a moving wave of compressions and expansions of the gas. A compressed region must compress the gas next to it for the wave to move forward. The molecules in the compressed region must move into, or knock others into, the region next to them. The wave can't have arrived before the molecules do. So the speed of the wave cannot be larger than the speed of the molecules; the two speeds may be comparable.

13. Copy the table of speeds of sound and add to it the values of speeds of molecules calculated for helium, nitrogen and carbon dioxide. How do the two sets of speeds compare?

Effect of temperature

14. If the temperature of a gas falls from 300 K to 273 K, by what factor do you expect the root mean square speed of its molecules to change?

15. Do the speeds of sound shown in the table follow a similar pattern?

Practical advice

These questions do not give a carefully argued reason why the speed of sound in a gas cannot exceed the speed of the molecules. But they show that there is a pattern, by taking molecules of different masses, and comparing two temperatures. Students see that despite large variations in the speeds in the different gases, the two speeds remain comparable. The questions give plenty of practice in calculation, manipulation of equations and reasoning about ratios.

The first questions (questions 1–9) are quick and simple, at the level of simple practice, and are suited to students of all abilities. The remaining questions take the level up to that at or a bit beyond the post-16 level examination.

You may have shown the rapid expansion of bromine into a vacuum, as evidence of the speed of molecules. Looked at a different way, this is a pressure shock wave (i.e. sound) travelling into the vacuum.

It may be helpful to have at the back of your mind the actual relation between the rms speed of molecules and the speed of sound. The speed of sound is

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

and since

$$P = \frac{1}{3}\rho\overline{c^2}$$

then

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

Thus the ratio of the rms speed to the speed of sound is

$$\sqrt{\frac{3}{\gamma}}$$

If for example $\gamma = 1.4$, the speed of molecules is approximately 46% greater than the speed of sound.

Alternative approaches

A more qualitative discussion could be better for less able candidates.

Social and human context

The theory of the speed of sound was worked out long before anyone had an idea of the speeds of molecules.

Answers and worked solutions

1. Helium, at 972.5 m s^{-1} at 273 K and 1019 m s^{-1} at 300 K.
2. Helium at 4 g mol^{-1}.
3. Carbon dioxide, at 257.4 m s^{-1} at 273 K and 269.8 m s^{-1} at 300 K.
4. Carbon dioxide, at 44 g mol^{-1}.

5. The speed of sound is larger in all cases at the higher temperature 300 K.

6. From the kinetic theory, the kinetic energy and so the speed of the molecules will be higher at the higher temperature, 300 K.

7.

$$Nkt = \tfrac{1}{3}Nm\overline{v^2}.$$

Divide both sides by N giving:

$$kt = \tfrac{1}{3}m\overline{v^2}.$$

Multiply both sides by 3 and rearrange, obtaining:

$$\overline{v^2} = \frac{3kT}{m}.$$

8.

$$m = \frac{4 \times 10^{-3}\ \text{kg mol}^{-1}}{6.02 \times 10^{23}\ \text{mol}^{-1}} = 6.64 \times 10^{-27}\ \text{kg}$$

9.

$$\overline{v^2} = \frac{3 \times (1.38 \times 10^{-23}\ \text{J K}^{-1}) \times 300\ \text{K}}{6.64 \times 10^{-27}\ \text{kg}} = 1.87 \times 10^{6}\ \text{m}^2\ \text{s}^{-2}$$

whence taking the square root the rms speed is 1370 m s^{-1}.

10. Since

$$\overline{v^2} = \frac{3kT}{m}$$

then the ratio of the mean square speeds is 4 / 28.

11. The rms speed for nitrogen will be √ (4 / 28) = 0.378 of the rms speed for helium, giving a speed of 0.378 × 1370 m s^{-1} = 517 m s^{-1}.

12. The factor is now √ (4 / 44) = 0.301 giving a speed of 0.301 × 1370 m s^{-1} = 413 m s^{-1}.

13. The table is now:

Gas	**Molar mass / g**	**Speed of sound at 273 K / m s^{-1}**	**Speed of sound at 300 K / m s^{-1}**	**rms speed of molecules at 300 K / m s^{-1}**
Helium	4	972.5	1019	1370
Nitrogen	28	337.0	355.5	517
Carbon dioxide	44	257.4	269.8	413

14. The squares of the speeds are proportional to the temperature, so if the temperature falls from 300 K to 273 K the speeds fall in the ratio √ (273 / 300) = 0.954.

15. Yes. The ratios 972.5 / 1019, 337.0 / 355.5 and 257.4 / 269.8 are all in the ratio 0.954 approximately.

External reference

This activity is taken from Advancing Physics chapter 13, 100S