Episode 603: Kinetic model of an ideal gas

This episode relates the gas laws to the behaviour of the particles of a gas.

Summary

Discussion and demonstration: explaining pressure in terms of particles. (15 minutes)

Discussion: deriving an equation for the pressure of a gas. (30 minutes)

Discussion: the link between KE and pressure. (20 minutes)

Worked example and student questions: Calculating molecular speeds. (20 minutes)

Discussion: internal energy of a gas. (15 minutes)

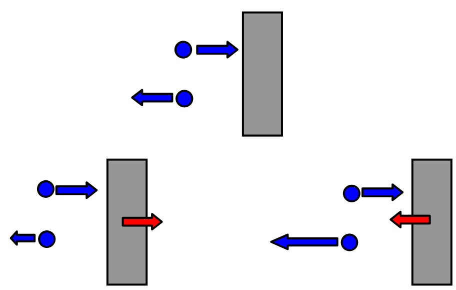
Discussion + demonstration:

Explaining pressure in terms of particles

In the previous episode, we looked at the macroscopic behaviour of a gas, in terms of its temperature, pressure and volume. Now we can go on to relate this behaviour to the underlying microscopic behaviour of the particles of which the gas is made.

Remind your students of the description of pressure arising from bombardment by particles of the walls of the container. Ask how the pressure will change if:

- the particles move faster (i.e. higher T)?
- the particles have greater mass m?
- there are more particles?



(All of these will result in greater pressure. You may wish to point out at this stage that increasing v has two effects: a greater force on impact, and more frequent impacts.)

You can usefully demonstrate the heating effect when the air in a bicycle pump is compressed.

TAP 603 -1: Warming up a gas by speeding up its particles

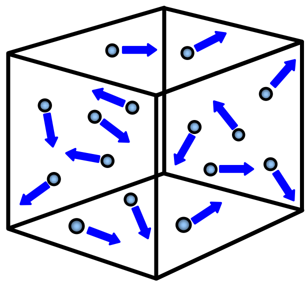
Discussion:

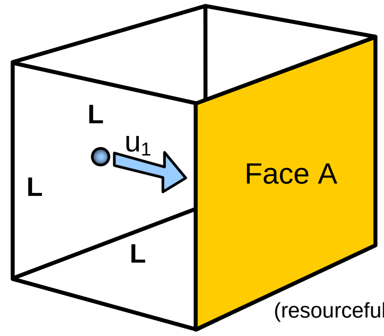
Deriving an equation for the pressure of a gas

The discussion above will prepare your students for the derivation and use of the equation for the pressure of an ideal gas, $pV = \frac{1}{3}Nm\overline{c^2}$ where *N* is the total number of molecules in the volume V, and the bar indicates an average. Check whether your specification requires the derivation. It is covered in most of the major texts. Even if the derivation is not required, you will have to explain the terms used, and show that the equation is plausible. Point out that the quantities on the left are macroscopic, while those on the right are microscopic.

Stress the underlying assumptions of the model. The gas:

1. has zero volume at zero temperature so the volume of the actual molecules is negligible.
2. has zero pressure at zero temperature so thermal energy is its source of kinetic energy.
3. have atoms or molecules which behave as elastic spheres with no long-range intermolecular forces.





(resourcefulphysics.org)

There are a number of points to look out for in the derivation. Students will need convincing about the idea of averaging the *square* of a velocity; the average velocity is zero, because particles with opposite velocities cancel out. The change in momentum is 2mv when an atom rebounds. The square of the velocity arises because *v* increases both the momentum change and the frequency of impact. We can average a series of impulses into a smooth net force to give an impulse Ft. All of this will rely on having covered momentum and impulse thoroughly.

If necessary try a demonstration to help convince the students. Balls strike a force sensor at increasing frequencies and with increasing velocity. The force sensor records the pressure which results.

TAP 603-2: One collision: many collisions

Discussion:

The link between KE and pressure

You can now compare the expression $PV = \frac{1}{3}Nm\overline{c^2}$ with the ideal gas law to show the

equivalence between temperature and the average KE per molecule. This idea follows directly from a statistical analysis of elastic collisions but the maths involved is too advanced for

the post-16 level. However the idea may be demonstrated by computer models that have programmed into them only the laws of dynamics for collisions, and nothing about thermodynamics. Such programs demonstrate the plausibility of the idea and are a powerful visual stimulus.

TAP 603-3: Kinetic theory applets

If your specification requires it, make that link formal with $\frac{1}{2}mv^2 = \frac{3}{2}kT$ where *k* is Boltzmann's

Constant, and $k=R/N_A$.

Worked example + student questions:

Calculating molecular speeds

This equation allows us to calculate mean molecular speeds, which should be done as a worked example. Calculate the rms speed for oxygen molecules at room temperature; ask the class to repeat the calculation for nitrogen, or for other molecules in air. They will find that the lighter the molecule, the faster its particles move.

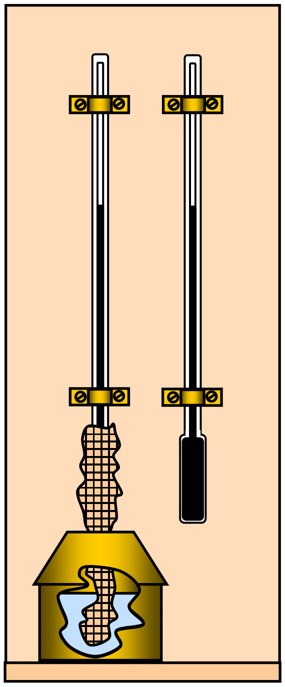
Compare these speeds with the speed of sound in air (~330 ms^{-1}), and with the Earth's escape velocity (11 kms^{-1}). You could also draw a comparison with familiar diffusion rates (e.g. speed with which a stink bomb is detected), leading to the idea of collisions and mean free path, which need not be entered into too deeply. The following question would be excellent to work through, leaving students to do some parts, and helping them through trickier concepts such as ratios.

TAP 603-4: Speed of sound and speed of molecules

Discussion:

Internal energy of a gas

You are now in a position to talk about *internal energy* as the kinetic and potential energy of the molecules. Potential has to be in there because energy must be supplied to melt a solid or boil a liquid; this does not become the kinetic energy of the particles, as the temperature stays constant. Another key idea here is that the energy calculated from the temperature using (3/2) kT = (1/2) mv^2 is an average and, therefore, there must be a distribution: some particles will have greater than this and some less. This leads to explanations of everything from evaporation to chemical reaction rates.



(resourcefulphysics.org)

All of the above are quite subtle, sophisticated ideas that require the students to absorb definitions of closely related concepts. As such it cannot be rushed, and discussion should be spaced out with examples such as:

1. Thermal energy flowing into a liquid initially raising the temperature, then boiling it, then heating it as a gas. All stages increase internal energy.
2. Joule's paddle wheel experiment where instead of heating increasing the internal energy, mechanical working is converted directly to internal energy. The temperature is raised and the *equivalence* of heating and doing mechanical work as two means of transferring energy is demonstrated. This is leading towards the first law of thermodynamics.
3. Cooling something down by evaporation. Compare when it is on a surface that then supplies further heat energy to keep the temperature constant (people sweat to lose heat energy) and evaporation when there is no such reheating, so that the liquid cools (puddles evaporate away totally). A drop of isopropyl alcohol on the back of the hand demonstrates the cooling effect clearly – it feels cold as it evaporates.

TAP 603 -1: Warming up a gas by speeding up its particles

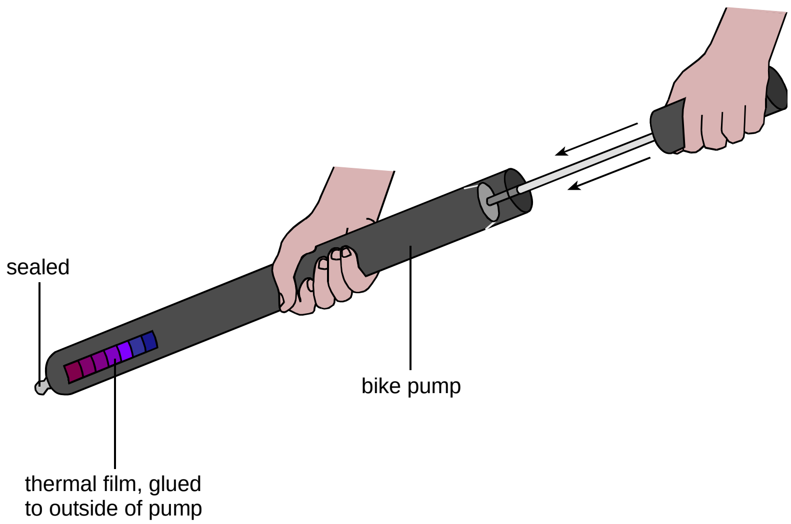
Arranging the molecules

Molecules move at random; by altering their environment you can get them to do useful things.

If you squash up a gas, pushing molecules into a restricted space, their average kinetic energy increases. You have to push the gas into shape, thus knocking into the molecules, which increases their speed.

You will need

- ✓ aluminium bodied bike pump with valve sealed / washer in normal position
- ✓ thermal film



What to do

1. Fully extend the pump and block the hole at the bottom with a close-fitting bolt and PTFE tape. Attach the thermal film to the sides of the cylinder near the bolt.

2. Make sure that the bicycle pump is cool. The temperature should be at the bottom of the range to which the thermal film will respond.

3. Fully extend the pump and squash the air up rather suddenly with one good push. Leave the piston at the position of maximum compression.

4. Watch for the temperature rise of the cylinder, as shown by the thermal film attached to it.

Practical advice

This is another simple experiment that can be explained using the kinetic theory of gases. Ask students to explain what an increase in temperature of a gas tells us about the particles of the gas. Follow this up by asking for explanations of the increased particle speed. It may take some time before the class is happy that collisions with the approaching piston increase the velocities of the gas particles. (Consider the momentum changes of an air particle as it rebounds from the approaching piston.)

Imagine the molecules engaged in violent collisions with the piston as it is closed rapidly. These collisions increase the average energy of the molecules, since they have more momentum (on average) when they leave the piston than when they arrive.

Technician's notes

An aluminium-bodied bicycle pump is essential, as is a well-oiled piston. The valve end can be sealed with an appropriately threaded bolt, plumbers PTFE tape, grease and a nylon washer. The thermal film should be fixed to the end furthest from the handle, where the warmed gas will reside, once it is compressed. For a similar reason it is advisable to move the handgrip on the barrel of the pump to the top third of the pump, as close to the plunger handle as is comfortable, to avoid thermal contamination from the users hands.

Thermochromic film is available from Middlesex University teaching resources http://www.mutr.co.uk/ as "thermocolour sheet".

Alternative approaches

Repetitive bending of a paperclip, or stretching a rubber band, leads to tangible heating. Rubbing on a sheet of thermal film produces a more spectacular, but less tangible, demonstration of the transfer of mechanical energy to thermal energy. Cooling can be done by rupturing a CO_2 drinks cylinder, or by allowing the contents of a camping gas cylinder to vent (take care!), provided that you note that at least some of the cooling in this case comes from pulling the molecules apart.

External reference

This activity is taken from http://www.practicalphysics.org/ which is an adaptation of Advancing Physics chapter 13, 230P

TAP 603-2: One collision: many collisions

A few molecules: lots of molecules

To change the momentum of a single molecule in collision with the walls of a container requires a force acting for a short time. Many molecules colliding with a surface produce a continual force, interpreted as a steady pressure. This arises from a continual rate of change of momentum, happening over a fixed area.

You will need

- ✓ force sensor connected to a fast data capture display device
- ✓ flexible plastic track
- ✓ five tennis balls
- ✓ bucket
- ✓ rubber tubing
- ✓ Hoffman clip
- ✓ access to water
- ✓ access to a gas tap

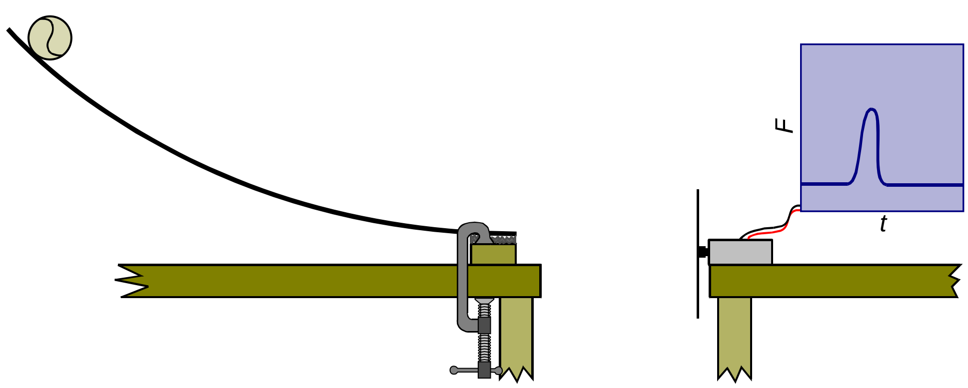
or

- ✓ access to pressurised air

Setting up

You will be looking at a force sensor linked up to a computer screen, recording the force exerted on a small plate about 5000 times a second. A variety of particles, from tennis balls to air molecules, will be fired at this plate in turn.

A single collision

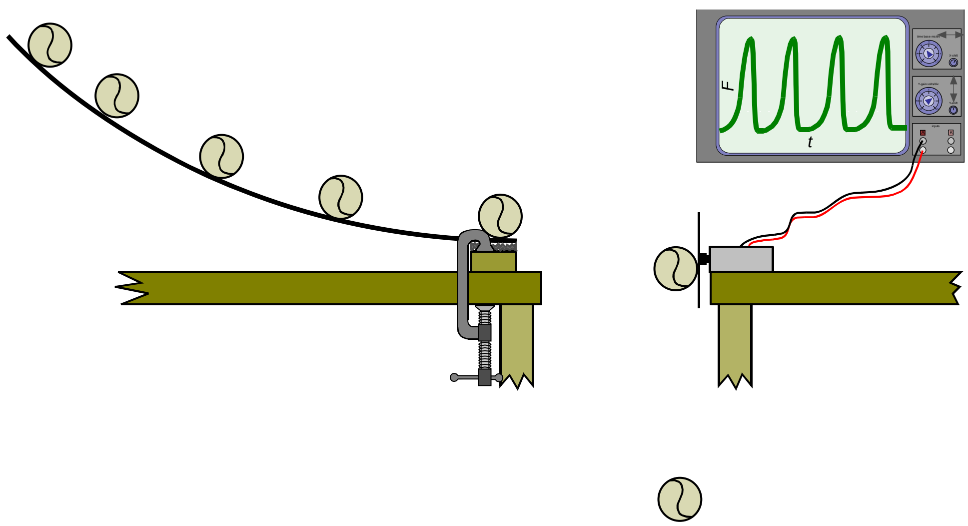


The ball trundles down the track into the force sensor. Here it needs to bounce, not to stick. Can you see why?

You can simulate ‘warming the gas’ by releasing the ball from further up the hill.

Some things to try:

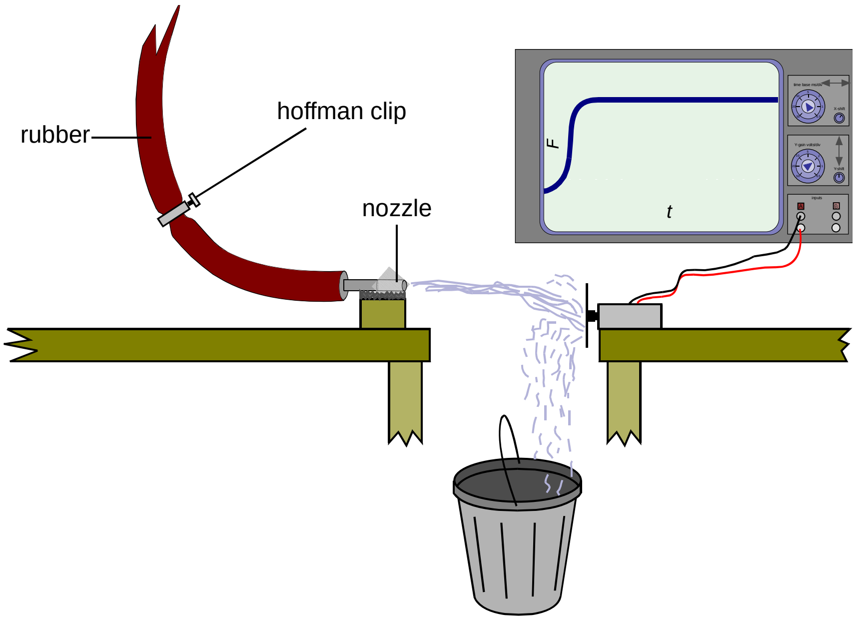
1. Measuring the peak force.
2. Relating the peak force to the speed of the ball.
3. Relating the momentum before the collision to the area under the force / time curve.
4. Many collisions. Now look at streams of balls, the number arriving per second gradually increasing.



Look out for the blips of force smearing out into a steady value, as you look over time scales that are long compared with the interval between collisions.

Fluid streams

Now look at streams of molecules producing forces that seem to be completely steady.



Things to think about and perhaps try:

1. Calculating the number of molecules hitting the force sensor per second.
2. Calculating the average interval between these collisions.
3. Comparing the rate of change of momentum with the force exerted.

You have seen

1. How many collisions a second produce a steady force.
2. How to relate the rate of change of momentum of the colliding particles to the force exerted.

Practical advice

You will need a force sensor capable of taking 5000 or so readings a second, and able to discriminate to 1N or so, in order to make this experiment work as written. A cheap substitute is to mount an old spring-calibrated set of kitchen scales horizontally, so as to measure forces in the horizontal plane, in place of the force sensor. This is unlikely to produce a discernible reading for the stream of air.

Tennis balls are preferred to harder balls, such a marbles or ball bearings, as they confer a degree of damping on the collision with the force sensor.

A single collision: Try to get a well defined F / t plot generated from a single collision with the force sensor. You might like to relate increasing velocity of impact to the changes in the F / t plot.

Many collisions: Now release a well spaced stream of tennis balls down the ramp, performing several runs, gradually reducing the interval between releasing the balls on each run, each time looking at the pulses of force. As the interval is reduced, encourage students to see an increasing smoothing of the force over time. You might rig up a vibration generator to release the balls at regular intervals, or try changing the mass or velocity of the balls. There are plenty of discussions that can arise from this experiment, all assisting with the fundamentals of the kinetic theory of gases.

Fluid streams: Here the bucket will be needed. You might carefully direct a stream of water at a plate connected to the force sensor (not over the force sensor!) and use the bucket to collect the water – so giving a mass per second. Knowing the diameter of the jet enables you to calculate the impact velocity – or constant-head apparatus and conservation of energy might give you a value. What happens here is open to development, according to the aptitude and interest of the class. Measuring these quantities should be an exercise in skilful manipulation of apparatus, and might be set as a challenge to the more able to pursue further and then present to the class. Or it might provide a starting point for an investigation.

A stream of air will also provide a measurable force on some force sensors (try it first!). Adapt the means of providing a controlled flow of air to suit local circumstances.

You might ask students to recall listening to rain on a roof, particularly a glass roof, and ask the following question: ‘How does this phenomenon illustrate these ideas?’.

Alternative approaches

You can replace all the high-tech measurements with a jug full of marbles poured over a top pan spring balance. There are advantages to seeing pulses of force smeared out by the averaging action of the spring in the balance. These approaches are supplementary.

Social and human context

An imaginative leap similar to this one is necessary to see smooth macroscopic effects emerging from discrete microscopic events.

External reference

This activity is taken from Advancing Physics chapter 13, 90P

TAP 603- 3: Kinetic theory applets

There are a number of simulations on the internet. You will need to pick those that suit your specification and needs. Some suggestions are below: -

The following were operational in October 2005

Boyle's Law

http://www.chm.davidson.edu/ChemistryApplets/KineticMolecularTheory/PV.html

Effect of temperature and volume:

http://lectureonline.cl.msu.edu/~mmp/kap10/cd283.htm

Distribution of velocity:

http://comp.uark.edu/~jgeabana/mol_dyn/KinThI.html

Molecular Model of an ideal gas:

http://www.phy.ntnu.edu.tw/ntnujava/viewtopic.php?t=42

TAP 603- 4: Speed of sound and speed of molecules

Calculating and comparing speeds

Look at the values of the speed of sound in a gas and the speed of its molecules. You will find that they are comparable in size, with the speed of the molecules always a bit greater, and you can think about why this should be true.

Comparing speeds of sound

Here is a table of measured values of the speed of sound in three gases:

Gas	Molar mass / g	Speed of sound at 273 K / m s^{-1}	Speed of sound at 300 K / m s^{-1}
Helium	4	972.5	1019
Nitrogen	28	337.0	355.5
Carbon dioxide	44	257.4	269.8

1. Which gas has the highest speed of sound, at either temperature?

2. Which gas has the least massive molecules?

3. Which gas has the lowest speed of sound, at either temperature?

4. Which gas has the most massive molecules?

5. At which temperature is the speed of sound the higher?

6. At which temperature are the molecules moving faster?

Comparing speeds of molecules

The next questions may suggest to you a reason for the pattern you have seen.

7. If there are *N* molecules in an ideal gas at temperature T, pressure P, volume V then

$$PV = NkT = \tfrac{1}{3} Nm\overline{v^2} ,$$

where the molecules have mass m and mean square speed $\overline{v^2}$, and k is the Boltzmann constant.

Show that the mean square speed is given by

$$\overline{v^2} = \frac{3kT}{m}.$$

8. Calculate the mass of a helium atom, given that 4 g (= 0.004 kg) of helium contains N = 6.02×10^{23} atoms.

9. Calculate the square root of the mean square speed for helium atoms, at 300 K, given that the Boltzmann constant k = 1.38×10^{-23} JK^{-1}.

10. The masses of nitrogen molecules and helium atoms are in the ratio 28 / 4. What should be the ratio of their mean square speeds at any given temperature?

11. Using the answer to question 9, predict the square root of the mean square speed (the rms speed) for nitrogen molecules at 300 K.

12. Repeat question 9 for carbon dioxide molecules.

Comparing speeds of sound and speeds of molecules

A sound wave in a gas consists of a moving wave of compressions and expansions of the gas. A compressed region must compress the gas next to it for the wave to move forward. The molecules in the compressed region must move into, or knock others into, the region next to them. The wave can't have arrived before the molecules do. So the speed of the wave cannot be larger than the speed of the molecules; the two speeds may be comparable.

13. Copy the table of speeds of sound and add to it the values of speeds of molecules calculated for helium, nitrogen and carbon dioxide. How do the two sets of speeds compare?

Effect of temperature

14. If the temperature of a gas falls from 300 K to 273 K, by what factor do you expect the root mean square speed of its molecules to change?

15. Do the speeds of sound shown in the table follow a similar pattern?

Practical advice

These questions do not give a carefully argued reason why the speed of sound in a gas cannot exceed the speed of the molecules. But they show that there is a pattern, by taking molecules of different masses, and comparing two temperatures. Students see that despite large variations in the speeds in the different gases, the two speeds remain comparable. The questions give plenty of practice in calculation, manipulation of equations and reasoning about ratios.

The first questions (questions 1–9) are quick and simple, at the level of simple practice, and are suited to students of all abilities. The remaining questions take the level up to that at or a bit beyond the post-16 level examination.

You may have shown the rapid expansion of bromine into a vacuum, as evidence of the speed of molecules. Looked at a different way, this is a pressure shock wave (i.e. sound) travelling into the vacuum.

It may be helpful to have at the back of your mind the actual relation between the rms speed of molecules and the speed of sound. The speed of sound is

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

and since

$$P = \frac{1}{3}\rho\overline{c^2}$$

then

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

Thus the ratio of the rms speed to the speed of sound is

$$\sqrt{\frac{3}{\gamma}}$$

If for example $\gamma = 1.4$, the speed of molecules is approximately 46% greater than the speed of sound.

Alternative approaches

A more qualitative discussion could be better for less able candidates.

Social and human context

The theory of the speed of sound was worked out long before anyone had an idea of the speeds of molecules.

Answers and worked solutions

1. Helium, at 972.5 m s^{-1} at 273 K and 1019 m s^{-1} at 300 K.
2. Helium at 4 g mol^{-1}.

3. Carbon dioxide, at 257.4 m s^{-1} at 273 K and 269.8 m s^{-1} at 300 K.

4. Carbon dioxide, at 44 g mol^{-1}.

5. The speed of sound is larger in all cases at the higher temperature 300 K.

6. From the kinetic theory, the kinetic energy and so the speed of the molecules will be higher at the higher temperature, 300 K.

7.

$$Nkt = \tfrac{1}{3} Nm\overline{v^2}.$$

Divide both sides by N giving:

$$kt = \tfrac{1}{3} m\overline{v^2}.$$

Multiply both sides by 3 and rearrange, obtaining:

$$\overline{v^2} = \frac{3kT}{m}.$$

8.

$$m = \frac{4 \times 10^{-3}\ \text{kg mol}^{-1}}{6.02 \times 10^{23}\ \text{mol}^{-1}} = 6.64 \times 10^{-27}\ \text{kg}$$

9.

$$\overline{v^2} = \frac{3 \times (1.38 \times 10^{-23}\ \text{J K}^{-1}) \times 300\ \text{K}}{6.64 \times 10^{-27}\ \text{kg}} = 1.87 \times 10^6\ \text{m}^2\ \text{s}^{-2}$$

whence taking the square root the rms speed is 1370 m s^{-1}.

10. Since

$$\overline{v^2} = \frac{3kT}{m}$$

then the ratio of the mean square speeds is 4 / 28.

11. The rms speed for nitrogen will be √ (4 / 28) = 0.378 of the rms speed for helium, giving a speed of 0.378 × 1370 m s^{-1} = 517 m s^{-1}.

12. The factor is now √ (4 / 44) = 0.301 giving a speed of 0.301 × 1370 m s^{-1} = 413 m s^{-1}.

13. The table is now:

Gas	**Molar mass / g**	**Speed of sound at 273 K / m s^{-1}**	**Speed of sound at 300 K / m s^{-1}**	**rms speed of molecules at 300 K / m s^{-1}**
Helium	4	972.5	1019	1370
Nitrogen	28	337.0	355.5	517
Carbon dioxide	44	257.4	269.8	413

14. The squares of the speeds are proportional to the temperature, so if the temperature falls from 300 K to 273 K the speeds fall in the ratio √ (273 / 300) = 0.954.

15. Yes. The ratios 972.5 / 1019, 337.0 / 355.5 and 257.4 / 269.8 are all in the ratio 0.954 approximately.

External reference

This activity is taken from Advancing Physics chapter 13, 100S