Episode 121: EMF and internal resistance

The starting point for the theory can be either Kirchhoff's second law or conservation of energy in the circuit (the same thing really) but a general discussion based on the circuit diagram below should use a variety of approaches.

Summary

Discussion: Deriving an equation (15 minutes)

Discussion: Practical effects of internal resistance. (10 minutes)

Student questions: Internal resistance of a power supply. (20 minutes)

**Student experiment: Measuring internal resistance and emf (45 minutes)**

Student questions: Practice questions. (30 minutes)

Discussion: More about the practical importance of internal resistance. (10 minutes)

Discussion:

Deriving an equation

There are three ways to arrive at the equation relating emf, terminal pd, current and internal resistance. It is worth discussing all three, to show their equivalence. The order you take will depend on the approach used previously with the class:
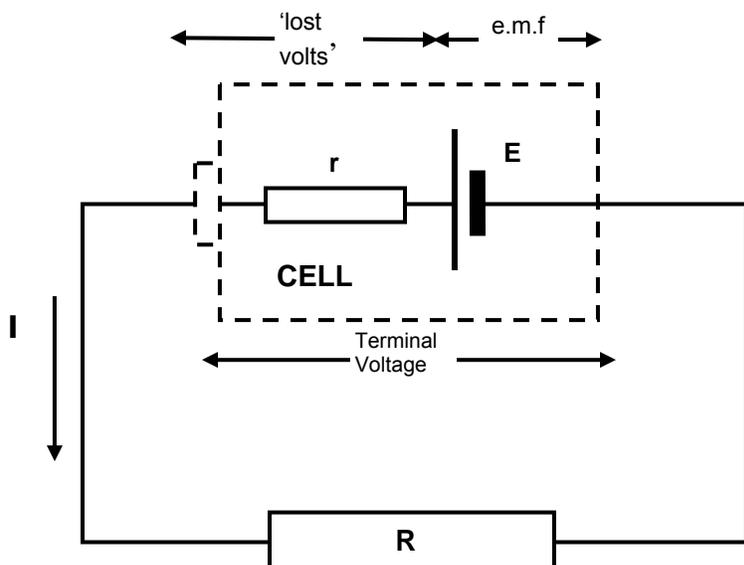
1.   Kirchhoff's 2nd Law: As charge goes around the circuit the sum of e.m.f s must equal the sum of voltage drops leading to:

E = I R + I r

The terminal voltage is equal to I R so this can be rearranged to give:

V = E − I r

and interpreted as terminal voltage = emf – 'lost volts'

2. Energy is conserved. Imagine a unit of charge, *Q*, moving around the circuit:

Q E = Q I R + Q I r

This leads to the same equations as in (1) above.

3. Use Ohm's law with E 'driving' current through the combined resistance (R + r):

I = E / (R+r)

Multiplying throughout by (R+r) leads to the same equations and conclusions as in (1).

Discussion:

Practical effects of internal resistance

At this point it might be worth pausing to illustrate the effects. Take a car as an example. The headlamps are connected in parallel across a twelve-volt battery. The starter motor is also in parallel controlled by the ignition switch. Since the starter motor has a low resistance it demands a very high current (say 60 A). The battery itself has a low internal resistance (say 0.01 Ω). The headlamps themselves draw a much lower current. Ask them what happens when the engine is started (switch to starter motor closed for a short time). Look for an answer in general terms:

- sudden demand for more current

- large lost volts (around 0.01 Ω × 60 A = 6 V)

- terminal voltage drops to 12 V – 6 V = 6 V

- headlamps dim
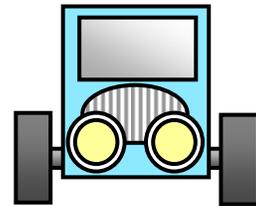
When the engine fires, the starter motor switch is opened and the
current drops. The terminal voltage rises and the headlamps return to normal. It's better to turn the headlamps off when starting the car.

As an aside, a lot of students seem to think the engine is powered by the battery! Point out that its main purpose while the engine is running is to provide the sparks for ignition and that while the car is driving the alternator continually recharges the battery, the energy for both headlamps and driving comes ultimately from the fuel that is burnt (since the car has to work a little bit harder to turn the alternator).

Student questions:

Internal resistance of a power supply

Some simple questions about the internal resistance of a power supply.

TAP 121-1: Internal resistance of power supplies

Student experiment:

Measuring internal resistance and emf

There are two experiments here, in which students determine the emf E and internal resistance r of cells - one involving a potato cell (leading to a high internal resistance) and one involving a

normal C cell (much lower internal resistance). You could get them to do both or ask some students to do one and some the other. Beware that, if you use an alkaline, high power C cell, it will run down quickly when there is a low load resistance, so you are advised to use cheap, low power cells which polarise quickly, they will depolarise over night. An alternative is to construct an artificial cell with a larger internal resistance by adding a higher series resistance (e.g. 100 Ω) to a standard cell.

TAP 121-2: Internal resistance of a source of emf

TAP 121-3: Internal resistance of a C cell

To determine E and r from the experimental results, there are various approaches. The simplest is to measure terminal voltage (V) and current (I) and to plot V against I. This gives an intercept at V = E on the *y*-axis and has a gradient of –r.

Student questions:

Practice questions

Questions on emf and internal resistance.

TAP 121-4: Questions on emf and internal resistance

Discussion:

More about the practical importance of internal resistance

Sometimes it is desirable to have a high internal resistance. Ask the class what happens if a

1.5 V cell is shorted - i.e. its terminals are connected together by a wire of zero resistance? Some might think I = V / R with $R = 0$ should mean that an infinite current would flow (limited by other physical factors!)

Remind them of the internal resistance r. This limits the cell to a maximum (short-circuit) current of:

I = E / r

We can use this to prevent EHT supplies giving the user an unpleasant shock. Take an EHT power supply off the shelf and show the connections for the series 'internal' resistance. It is usually 5 MΩ.

These supplies are designed to provide a high voltage to a high resistance load (e.g. cathode ray tube) but if the terminals or wires connected to them were accidentally touched this could provide a nasty shock (lower resistance in the load and higher current). One way to deal with this is to connect a large resistance in series with the output (positive) terminal. If the terminals are shorted (e.g. by contact through a person) the current drawn is limited to I = E / r. A typical EHT supply (up to 5000 V) is protected by a 5 MΩ resistor so the maximum current if shorted is just 1 mA. That shouldn't kill you! Be aware however that HT supplies (0-300 V) have a much lower internal resistance, and could kill you, so special shrouded leads should be used.

EHT supplies often have a further 'safety resistor' (e.g. 10 MΩ) to reduce the maximum current still further. This resistor can be by-passed when necessary. No school EHT supply is allowed to provide more than 5 mA.

Potential (V)

E

r

R

I r

E

IR
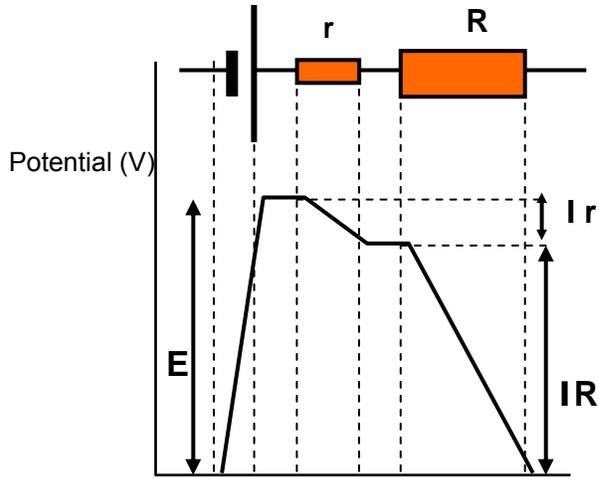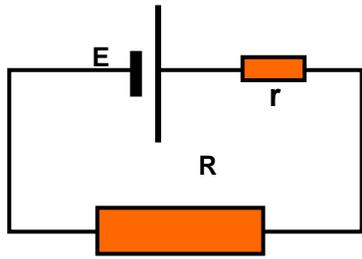
(resourcefulphysics.org)

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TAP 121-1: Internal resistance of power supplies

Answer the following questions for practice in making calculations about the internal resistance of power supplies.

Torch batteries, car batteries, EHT supplies and solar cells

1. A typical hand-held torch runs off two 1.5 V cells, yet has a lamp rated at 2.5 V, 0.5 A. Explain how the potential difference across the lamp can actually be 2.5 V as rated. What is the internal resistance of each cell, supposing them to be identical?

2. A typical car battery has an emf of 12 V, and must provide a current of 80 A to the starter motor. Why must the car battery have a very low internal resistance? If the internal resistance is 0.05 Ω, find the potential difference across this internal resistance when the starter motor is running. Why is starting the car with the headlights on likely to affect their brightness?

3. Some school laboratories have EHT (Extra High Tension) power packs giving up to

    3000 V. For safety, they are provided with a 50 MΩ resistor in series with the supply. What is the maximum current able to be drawn from the supply? Approximately what potential difference would there be across a torch bulb connected across such a supply?

4. A student experimenting with a solar cell connects a 1000 Ω voltmeter across it and observes a potential difference of 1.0 V. Using a different, extremely high resistance digital voltmeter, the reading is larger, 1.2 V. Why the difference? What is the internal resistance of the solar cell?

Practical Advice

These are intended to be simple practice questions. It is helpful to remember that the internal resistance of a cell is not likely to remain constant as the cell is used, and that other effects such as polarization of the cell also affect the pd obtained from it.

Alternative Approaches

It is useful for the class to check that torch bulbs are very commonly rated at less than the emf of the dry batteries they use. Taking various examples, the range of values the makers of torches expect for the internal resistance of dry cells can be estimated.

It is also useful to review power supplies available in the laboratory, looking to see which must have low internal resistance and which normally need extra protective resistance added.

Social and Human Context

Dry cells power all sorts of portable equipment besides torches. Some, such as television or video control handsets, can run for years on one set of batteries. Others, such as palm-top computers and 'organizers', use up batteries very quickly.

Answers and Worked Solutions

1. The two 1.5 V cells provide an emf of 3 V in series. If the current flowing is 0.5 A as stated, then for the potential difference across the internal resistance to be 0.5 V (that is, 3 V–2.5 V) the internal resistance of the cells combined would need to be 1 Ω. The cells are in series so the resistance of each is 0.5 Ω.

2. The battery must have a low internal resistance so as to be able to deliver a current of 80 A from an emf of only 12 V. If the internal resistance is 0.05 Ω then the potential difference across this with a current of 80 A flowing is 4 V. Thus the potential difference across the 12 V battery drops to 8 V. This is a big enough change to dim headlights rated at 12 V.

3. The maximum current is 60 μA. A torch bulb has a resistance of only a few ohms, so connected across such a supply the potential difference across it would be very near to zero, with a current of only 60 μA through it.

4. The 1000 Ω voltmeter draws a current from the cell, of 1 mA when it reads 1.0 V. If the cell has internal resistance some of its emf will be used in driving the current through the cell. A voltmeter with very high resistance draws very little current, and reads nearer to the emf of the cell. If the emf is 1.2 V then 0.2 V is used in driving the current of

   1 mA through the internal resistance, which is therefore 200 Ω.

External References

This activity is taken from Advancing Physics Chapter 2, 220S

TAP 121-2: Internal resistance of a source of emf

Some of the energy given to charges by a cell (or other source of emf) is dissipated inside the cell itself, as the charges move through the cell (or other source of emf). What is left is available as a potential difference (energy per unit charge) across a circuit connected to the cell. If the emf of the source is E, and its internal resistance is r, then when a current I flows the potential difference V is $V = E - I\,r$.

Collect this apparatus

✓   2 digital multimeters

✓   potato

✓   0.5 cm × 2 cm copper sheet, 0.5 cm × 2 cm zinc sheet

✓   2 pairs of crocodile clips

✓   resistance substitution box

✓   5 × 4 mm leads

Getting, and making sense of, the data

First take a quick look. Then collect some detailed data that you can use to model the behaviour of the cell.

1.   Assemble your cell. Use the copper and zinc sheets as electrodes, inserting one in each end of the potato. Use crocodile clips to make connections to the circuit.

potato
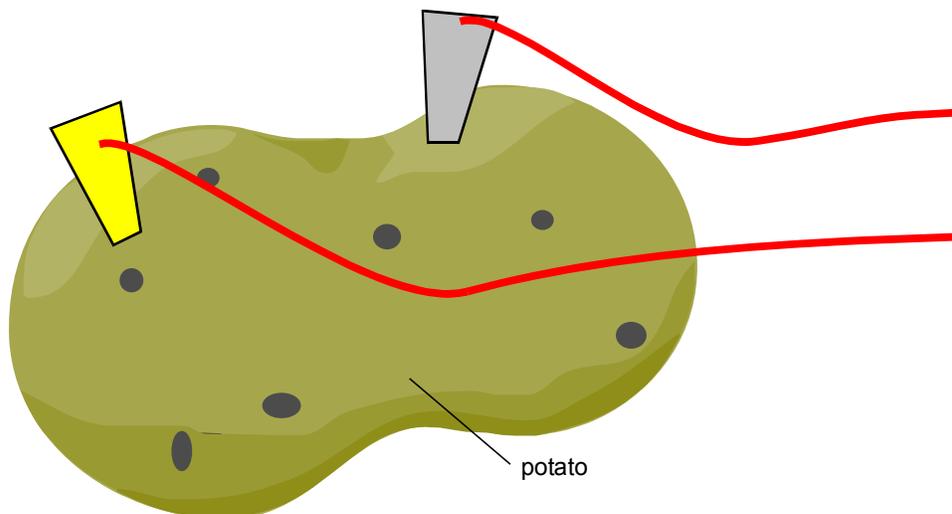
2.   Set up a circuit to measure the pd across the cell and the current drawn from the cell, initially with 4.7 kΩ as a load for the cell.
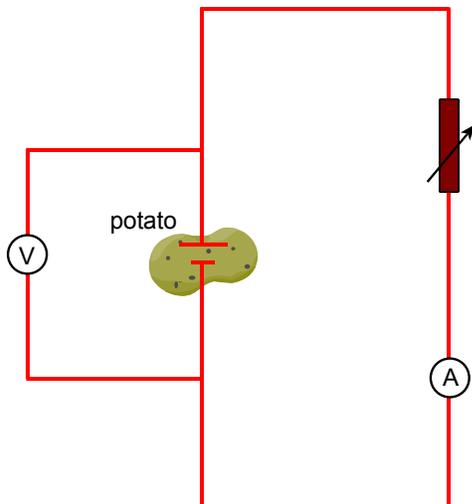
3. Alter the load resistance. Notice the changes to the current drawn and the pd supplied. Sketch a graph of pd / current to indicate the general trend.

4. Look back to the introduction above. Does your pattern seem likely to fit this description?

5. Now draw up a table of current and pd for a range of load resistances. You will need to be careful in selecting the values to use at both ends of the scale so that your measuring instruments can cope.

6. Plot a graph of V / I. Does it fit the pattern above?

Outcomes

1. You will recognise the drop of pd as a source supplies an increasing current.

2. You will be able to match this pattern to the description above.

Practical Advice

This is designed as a simple introduction to the phenomenon, followed by a more detailed look.

The advantage of using a potato is the path the current might take between the electrodes is transparent. The idea of resistance internal to the cell and energy dissipated there is easier to appreciate if it the cell is visible.

Alternative Approaches

You could make 'cells' with artificially high internal resistance by soldering a resistor in series with a standard dry cell.

Solar cells, cheaply available from surplus suppliers, make good sources with appreciable internal resistance. The internal resistance alters with incident illumination and thus some experimentation is needed, with a 48 W lamp, to get optimum conditions. Then you can use

50 mA fsd and 20 V fsd meters to get a good plot. A 16 W rheostat functions as a variable load. You are warned that solar cells are not uniform in performance and some fine-tuning may be necessary.

Solar cells can be purchased cheaply from recycling mail order suppliers and then mounted on plastic electrical ducting.

Social and Human Context

The dip in the brightness of car headlamps when the starter motor is used appears to be the most common experience to draw on.
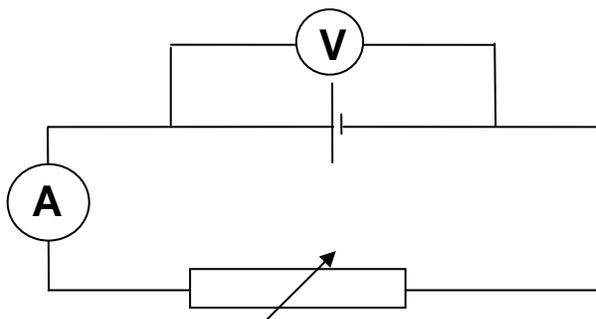
External References

This activity is taken from Advancing Physics Chapter 2, 240E

TAP 121-3: Internal resistance of a C cell

Apparatus required:

- ✓ C cell (possibly modified by the addition of a series resistor)
- ✓ Two multimeters (or an ammeter and voltmeter)
- ✓ Rheostat (approx. 10 Ω)
- ✓ Leads

Circuit:

Procedure

Start with the rheostat on its maximum resistance. Record V and I. Gradually reduce the rheostat to its lowest resistance (zero) measuring V and I a minimum of 7 times over the range. Don't leave the circuit connected for long when the resistance is low (current high) because this will run the cell down quickly.

Plot a graph of V against I. E is the intercept on the V axis. The gradient is $-r$.

Another alternative approach is to interface the experiment to a computer using a suitable package. If you do this you can collect current and voltage readings immediately as you sweep the rheostat across its range of values. Most packages allow you to plot the graph as you collect data so the students will see this plotted in real time.

Practical Advice

The experiment is easier for students to perform with an extra resistance added in series with the cell. 1 or 2 Ω is suitable, though 10 Ω could be used.

DO NOT use alkaline cells as the internal resistance is too low unless an extra resistor is used.

Make it clear that the 'cell' is the power source plus the resistor.

A zinc carbon cell will give around 5 A if shorted out (dependent on size). That from an alkaline cell is much larger, it is best to protect the cells with an external resistance. If nothing else the cell will last longer as it could be left shorted out by the student.

TAP 121- 4 Questions on EMF and Internal Resistance

1. A 9.0 V battery has an internal resistance of 12.0 Ω.

(a) What is the potential difference across its terminals when it is supplying a current of 50.0 mA?

(b) What is the maximum current this battery could supply?

(c) Draw a sketch graph to show how the terminal potential difference varies with the current supplied if the internal resistance remains constant. How could the internal resistance be obtained from the graph?

2. A cell in a deaf aid supplies a current of 25.0 mA through a resistance of 400 Ω. When the wearer turns up the volume, the resistance is changed to 100 Ω and the current rises to 60 mA. What is the emf and internal resistance of the cell?

3. Explain why the headlamps of a car go dim when the starter motor is used.

4. A battery is connected in series with a variable resistor and an ammeter. When the resistance of the resistor is 10 Ω the current is 2.0 A. When the resistance is 5 Ω the current is 3.8 A. Find the emf and the internal resistance of the battery.

5. When a cell is connected directly across a high resistance voltmeter the reading is 1.50 V. When the cell is shorted through a low resistance ammeter the current is 2.5 A. What is the emf and internal resistance of the cell?

6. You are supplied with 6 identical dry cells, each of emf 1.5 V and internal resistance 0.3 Ω. What are the overall emf and internal resistance when:

(a) the cells are connected in parallel?
(b) the cells are connected in series?
(c) they are connected in three groups, each of two cells in series, and these groups are connected in parallel with one another?

Assume the polarity of all the cells in each arrangement is the same.

7. A 'potato' cell has emf 1.0 V and internal resistance 5000 Ω. Roughly how many of these cells in what arrangement would adequately light a 5 W, 6.0 V filament lamp?

Answers and Worked Solutions

1. (a) pd = E – I r = 9 – (50 x 10^{-3} x 12) = 8.4 V

   (b) Max current = E/r = 9 / 12 = 0.75 A

2. E = I(R +r)

    E = 25 x 10^{-3} (400 + r)   and   E = 60 x 10^{-3} (100 + r)

    So

    25 x 10^{-3} (400 + r) = 60 x 10^{-3} (100 + r) so r = 114.3 Ω

    E = 10 + (25 x 10^{-3} x 114.3) = 12.86 V

4. E = I(R +r)

    E = 2 (10 + r)   and   E = 3.8 (5 + r) so r= 0.56 Ω

    E = 20 + (2 x 0.56) = 21.1 V

5. E = 1.5 V

    E = I r so 1.5 V = 2.5 A r and r = 0.6 Ω

6. (a) 1.5 V, 0.05 Ω

   (b) 9 V, 1.8 Ω

   (c) 3V, 0.2 Ω

7. Required pd = 6 V and power = 5 W so current required = P/V = 5/6 = 0.83 A

    6 cells in series have an internal resistance of 6 x 5000 = 30000 Ω . If the total internal resistance is low, say 0.1 Ω then 6 - 0.083 = 5.92 V roughly which would light the lamp well, so 30,000/n = 0.1

    and n = number of combinations = 300,000 so total number of cells is 6 x 300,000 = 1,800,000

The answer depends on what you mean by adequately light. Would external pd of 5 V and total internal resistance 1 Ω be sufficient for example?