Tap 129- 6: Capacitors with the exponential equation

Practice in using the exponential equation

You will find it useful to be able to use the equation to calculate, for example, the pd across a capacitor after it has been discharging for some time. The following question give you practice in doing this.

A 10 F capacitor is charged to 5.0 V and then discharged through a 5 kΩ resistor.

1. Calculate the time constant for the circuit.

2. How much energy is stored in the capacitor when it is fully charged?

3. Calculate how long it takes for the pd across the capacitor to fall to 4.0 V.

4. How much energy will have been transferred from the capacitor during this process?

5. What will be the pd across the capacitor after 5.0×10^4 s?

6. Calculate the time taken for 50% of the capacitor's initial energy to go into heating in the resistor.

Practical advice

These questions involving solving problems using the equation may only be suitable for those aiming at higher grades, or the mathematically more adept.

Answers and worked solutions

1. 5.0×10^4 s
2. 125 J
3. About 3 hours
4. 45 J
5. 1.8(4) V
6. 1.7×10^4 s

Worked solutions

1.

$$\tau = (5 \times 10^3\ \Omega) \times 10\ \text{F} = 5 \times 10^4\ \text{s}$$

2.

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 10\ \text{F} \times (5.0\ \text{V})^2 = 125\ \text{J}$$

3.

$$Q = Q_0 \text{e}^{-t/RC} \quad \text{and} \quad Q = CV$$

so

$$V = V_0 \text{e}^{-t/RC}$$

$$4.0\ \text{V} = 5.0\ \text{V} \times \text{e}^{-t/(5 \times 10^4\ \text{s})}$$

gives

$t = 186$ min.

4.

Energy at 4.0 V

$$E = \tfrac{1}{2} \times 10\ \text{F} \times (4.0\ \text{V})^2 = 80\ \text{J}$$

so the energy lost is 45 J.

5.

$$V = 5.0\ \text{V} \times \text{e}^{-(5.0 \times 10^4\ \text{s})/(5.0 \times 10^4\ \text{s})} = \frac{5.0\ \text{V}}{\text{e}} = 1.8(4)\ \text{V}$$

6. For 50% energy:

$$V = \frac{5.0\ \text{V}}{\sqrt{2}} = 3.54\ \text{V}$$

so

$$3.54\ \text{V} = 5.0\ \text{V} \times \text{e}^{-t/(5 \times 10^4\ \text{s})}$$

which gives t = 1.73 x 10^4 s, or just under 5 hours.

External references

This activity is taken from Advancing Physics Chapter 10, 140S