

Episode 128: Energy stored by a capacitor

So far, we have not considered the question of energy stored by a charged capacitor. Take care; students need to distinguish clearly between charge and energy stored.

Summary

Demonstration: Energy transformations. (15 minutes)

Discussion: Calculating energy stored. (15 minutes)

Worked example: Energy stored. (10 minutes)

Student experiment: Energy stored – two alternatives. (20 minutes)

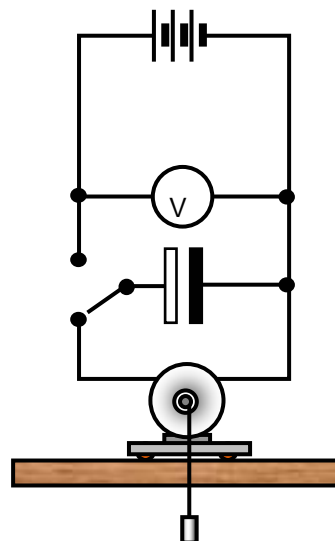
Student questions: Calculations on the energy formula. (30 minutes)

Demonstration:

Energy transformations

The idea that a capacitor is a store of electrical energy may have already emerged in previous sections but it can be made clear by using the energy stored in a capacitor to lift a weight attached to a small motor. The energy transfer process is not very efficient but it should be possible to show that a larger pd (or a higher capacitance) lifts the weight further. Hence energy stored depends on both C and V .

Emphasise the link between work and energy. How do we know that the charged capacitor stores energy? (It can do work on the load.) How did the capacitor gain its energy store? (The power supply did work on the charges in charging the capacitor.)



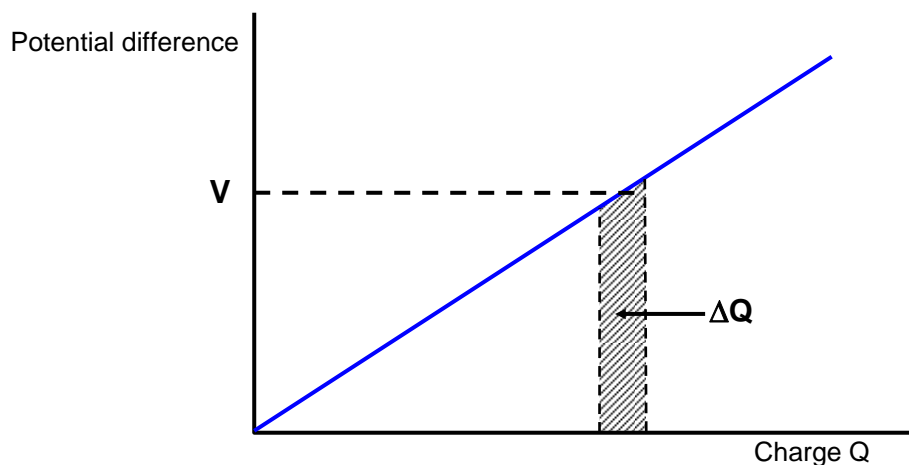
TAP 128.1: Using a capacitor to lift a weight.

Discussion:

Calculating energy stored

Having seen that the energy depends on the voltage, there are several approaches which lead to the relationship for the energy stored. Start with a reminder of the idea that 'joules = coulombs \times volts'.

The simplest argument is that with a pd V , a capacitor C will store charge Q , but the energy stored is not $Q \times V$. Why not? (As the capacitor charges, both Q and V increase so we have not moved all the charge with a pd of V across the capacitor.)



What does this graph tell us? At first, it is easy to push charge on to the capacitor, as there is no charge there to repel it. As the charge stored increases, there is more repulsion and it is harder (more work must be done) to push the next lot of charge on.

Can we make this quantitative? A first try says that the pd was on average $V/2$, so the energy transformed was $Q \times V/2$.

A more general approach says that in moving the charge ΔQ , the pd does not change significantly, so the energy transformed is $V \times \Delta Q$. But this is just the area of the narrow strip, so the total energy will be the triangular area under the graph.

i.e. Energy stored in the capacitor = $1/2 QV = 1/2 CV^2 = 1/2 Q^2/C$

If your pupils are strong mathematically, this summation can be replaced by integration.

Worked example: Energy stored

A $10 \mu\text{F}$ capacitor is charged to 20 V . How much energy is stored?

Emphasise how to choose the correct version of the equation, in this case:

Energy stored = $1/2 CV^2 = 2000 \mu\text{J}$

Ask you students to calculate the energy is stored at 10 V (i.e. at half the voltage). Answer:

$500 \mu\text{J}$, one quarter of the previous value, since it depends on V^2 .

Student Experiment: Energy stored – first alternative

The formula can be checked with either or both of the following experiments.

The first experiment is straightforward. It could be used as the basis of a demonstration in which you ask the pupils to suggest how many extra bulbs are required at each stage and how they should be connected.

TAP 128-2: How many bulbs will a capacitor light

Student Experiment: Energy stored second alternative

The second experiment needs more apparatus and time, and needs patience to obtain accurate measurements; it has benefit in terms of thinking about experimental design and systematic errors.

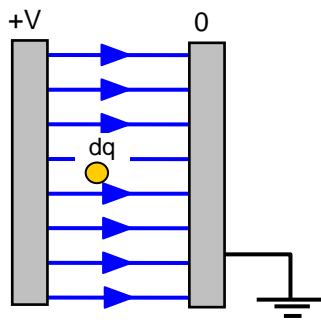
Tap 128-3: Energy stored by a capacitor

Student Questions: Calculations on the energy formula

These give practice with the energy formulae.

TAP 128-4: Energy stored in a capacitor

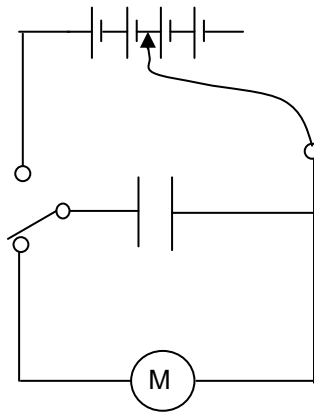
TAP 128-5: Energy to and from capacitors



TAP 128- 1: Using a capacitor to lift a weight

Requirements

- ✓ 10 000 μF capacitor (or larger if possible) working voltage $> 6\text{V}$
- ✓ small motor 3 – 6 V is ideal
- ✓ 4 \times 1.5 V cells (or dc supply)
- ✓ small weights (typically 20 g but it will depend on the motor)
- ✓ length of thin cotton thread
- ✓ leads



Tie a small mass to the motor spindle with a length of cotton so that the mass is just above the floor.

Find the voltage at which the motor is just able to lift the weight (this is usually well below the nominal voltage for the motor).

Using this voltage, charge the capacitor and then discharge it through the motor. The weight should be raised by a few centimetres. Increase the voltage used and compare the distance raised at low and high voltages.

[The efficiency of the motor is usually quite small ($\sim 10\%$) so quantitative work is best avoided.]

Tap 128- 2: How many bulbs will a capacitor light?

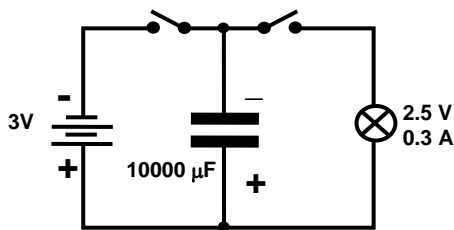
By charging a 10 000 μF capacitor to different voltages and discharging into various numbers of small light bulbs, suggest a relationship between capacitor voltage and the energy it stores.

Requirements

- ✓ 10 000 μF electrolytic capacitor
- ✓ Locktronics baseboard or similar
- ✓ m.e.s. bulb holders (9)
- ✓ m.e.s. 2.5 V 0.3A bulbs (9 – must be well matched)
- ✓ voltmeter (f.s.d. 10 V)
- ✓ l.t. variable voltage power supply or a set of 1.5 V cells to reach 9 V.
- ✓ connecting leads

Set up

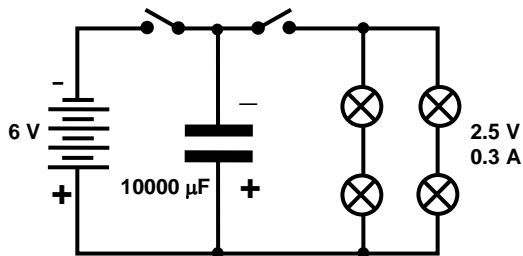
Connect up the circuit shown in the diagram below so that initially the 10 000 μF capacitor is being charged to 3 V.



Now discharge the capacitor into the single light bulb and observe its brightness.

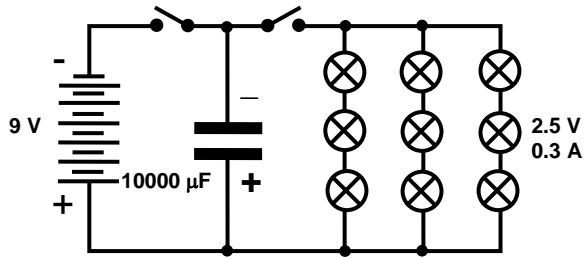
This is to be your 'standard illumination' which to compare the others with.

Charge the capacitor to 6 V and discharge into the four bulbs as shown below.



Observe the brightness of the bulbs. Are they lit about the same as the first?

Finally, charge the capacitor to 9 V and discharge into the nine bulbs as shown in the diagram below.



Observe the brightness of these bulbs compared with your initial standard brightness.

Suggest a relationship between the capacitor voltage and the energy stored.

Practical advice

You will need more than nine m.e.s. bulbs in order to get nine that are well matched.

To test for a match, connect all the bulbs in parallel on a board to a supply voltage of 2.5 V and select until all are of equal brightness.

Alternatively measure the m.e.s. bulb resistance with a multimeter and select bulbs with the same resistance.

External references

This activity is taken from Salters Horners Advanced Physics, A2, Medium is the Message, MDM Activity 23

TAP 128- 3: Energy stored in a capacitor and the potential difference across the plates.

Introduction

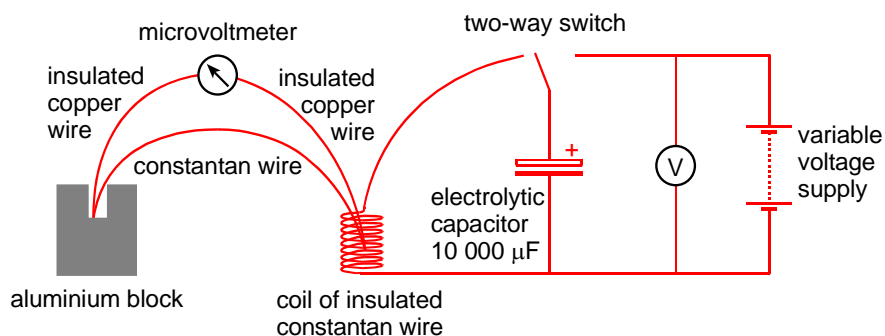
This demonstration is intended to make the link between the energy stored in a capacitor and the potential difference to which it has been charged. It relies on the heating effect of the current which flows when the capacitor is discharged resulting in a measurable rise in temperature. The rise in temperature is assumed to be proportional to the energy stored. It is possible to obtain a series of readings by charging the capacitor to different potential differences and determining the rise in temperature each time.

Requirements

- ✓ capacitor 10 000 μF , 30 V working
- ✓ resistance coil (about 1 m of 34 swg insulated eureka wire wound into a small coil which will tightly enclose the temperature probe/thermocouple), or thermally insulated carbon film resistor of a few ohms resistance.
- ✓ thermocouple, copper–constantan
- ✓ micro voltmeter
- ✓ spdt switch
- ✓ multimeter
- ✓ power supply, 0–25 V
- ✓ aluminium block
- ✓ leads, 4 mm

Set-up

1. Set up this apparatus.



Make sure that the capacitor is connected to the supply correctly otherwise it may suffer damage. You will need to use the thermocouple connected to a sensitive voltmeter, measuring in mV.

2. Adjust the output of the dc supply to (say) 5.0 V.

3. Connect the capacitor to the supply using the two-way switch so that it becomes charged. Record the reading on the voltmeter.
4. Discharge the capacitor through the resistance coil and record the reading on the sensitive voltmeter, the output from the thermocouple.
5. Charge the capacitor to a different potential difference and repeat the experiment. Obtain a series of values of potential difference V and sensitive voltmeter readings. Do not exceed the safe working voltage of the capacitor (30 V).
6. Plot a graph that will enable you to deduce a relationship between the energy stored in the capacitor and the potential difference across its plates.

You are assuming that the energy stored in the capacitor is equal to the change in internal energy of the resistance coil, which is proportional to the temperature rise of the coil. Note that the reading on the sensitive voltmeter is proportional to the rise in temperature of the thermocouple probe.

What you have learned

1. From the shape of your graph, you should be able to deduce the link between the pd across the capacitor and the energy stored in it, as measured by the temperature rise.

Practical advice

Care should be taken to ensure that electrolytic capacitors are connected with the correct polarity and that the working voltage is not exceeded.

It will be worth spending some time familiarising students with the thermoelectric effect. This provides an essential link to temperature rise and thus the energy stored by the capacitor. It is advisable to discuss in advance how to analyse the data recorded.

Be careful to shield the apparatus from draughts, as they affect the final temperature rise obtained. Ensure the 'cold junction' of the thermocouple is in thermal contact with the aluminium block – a few drops of glycerol may help, with a smack rubber bung to hold the wires still.

Try out the experiment first for yourself to decide whether or not it will yield quantitative information about the relationship $W = \frac{1}{2} C V^2$. If there is too much scatter in the points, there will be no justification for plotting a straight line graph. If it is not convincing quantitatively, it may still be worthwhile to carry out the experiment qualitatively.

The use of small bead resistor instead of the coil enables the thermocouple's hot junction to be taped securely to its surface. As stated, shielding from draughts is important, as the hot junction assembly has small thermal inertia.

The apparatus can be calibrated by delivering short timed pulses of current at a known voltage from the power supply, to calibrate the output, in terms of temperature rise, against a known energy input ItV . As such the exercise has to be treated as an extended experiment, which will be carried out over 2 practical sessions.

Alternative approaches

A preliminary demonstration to show how a discharging capacitor can briefly light a bulb could be useful. Large value capacitors, used as backup power supplies for memory (5 V working, 1.0 F) are also available, and can light LEDs for considerable lengths of time.

You might use a temperature probe for the main experiment if you have a suitable one with a sufficiently small thermal capacity.

External references

This activity is adapted from Advancing Physics, Chapter 10, 130E

TAP 128- 4: Energy stored in capacitors.

What to do

Work your way through the questions, preferably in the order in which they appear. The first few questions will give you some practice in using the equations relating to energy storage.

Questions

A 50 microfarad (μF) capacitor is charged to a pd of 60 V.

1. Calculate the charge on the capacitor.
2. Calculate the energy stored.
3. Calculate the energy stored when the pd is doubled to 120 V.
4. Compare your answers to questions 2 and 3. What does this tell you about the relationship between the energy stored by a capacitor and the pd to which it has been charged?
5. A 1000 μF capacitor is charged so that it stores 2.0 J of energy. Calculate the pd to which it has been charged.
6. The incomplete table below contains values of capacitance, charge, pd and energy for a series of charged capacitors. Carry out calculations and fill in the blanks in the table.

Capacitance	Charge	Potential difference	Energy
1000 μF		16 V	
10 mF	0.01 C		
1.0 F			100 J
	2.0 mC	5000 V	
		100 V	50 mJ
33 000 μF			2.0 J

A $1.0 \mu\text{F}$ capacitor is charged to a pd of 10 V.

7. Calculate the charge on the capacitor.

8. How much charge flowed through the battery during charging?

9. How many electrons flowed through the battery during charging?

10. Calculate the energy stored by the capacitor.

11. How much energy was transferred from the battery during the charging process?

12. (Rather harder) You should have found different answers for questions 10 and 11. Explain this difference.

Hints

1. You will have to do quite a bit of rearranging of equations.
6. Think of the definition of potential difference when you are trying to work out how much energy is involved in passing charge through a battery.

Practical advice

The range of questions is designed to give students practice in handling the relevant equations. It is advisable that students work through the questions more or less in the order they appear. Because there are three different equations relating to energy storage in capacitors, there are several instances where students have to choose which to use for their own convenience.

There are many pitfalls, not deliberate, into which students can fall because of difficulties with unit prefixes and with standard form. From this point of view, the questions provide some good practice in handling both of these issues.

If they have met only $E = \frac{1}{2} Q V$ but not the other versions, it may be necessary to spend some time with them developing 'variations on the theme'. Question 12 is an issue familiar to teachers but difficult for most students to grasp; they might need more help than is given in the hints. It is probably best left as extension work for students aiming for higher grades.

Answers and worked solutions

1. $3.0 \times 10^{-3} \text{ C}$
2. 0.090 J
3. 0.36 J
4. If the pd is doubled, energy stored goes up by factor of four; energy is proportional to the square of the pd
5. 63 V
- 6.

Capacitance	Charge	Potential difference	Energy
1000 μF	0.016 C	16 V	0.013 J
10 mF	0.01 C	1.0 V	5 mJ
1.0 F	14 C	14 V	100 J
$4.0 \times 10^{-7} \text{ F}$	2.0 mC	5000 V	5 J
10 μF	1.0mc	100 V	50 mJ
33 000 μF	0.36 C	11 V	2.0 J

7. $1.0 \times 10^{-5} \text{ C}$
8. $1.0 \times 10^{-5} \text{ C}$
9. 6.2×10^{13}
10. $5.0 \times 10^{-5} \text{ J}$
11. $10.0 \times 10^{-5} \text{ J}$
12. 50% of the energy from the battery is used to heat the connecting wires (warming the wires). If there were no resistance somewhere in the circuit we would have a non-physical situation of the system discharging in an infinitesimal time, with infinite currents.

In practice inductive effects would also come into play, even if we had a superconducting circuit. This raises the issue of boundary conditions- you can't have a non-physical system.

(See Physics Education July 2005 pages 370 and following.

For details how to obtain Physics Education see the IOP website.)

Worked solutions

1.

$$Q = CV = (50 \times 10^{-6} \text{ F}) \times 60 \text{ V} = 3.0 \times 10^{-3} \text{ C}$$

2.

$$E = \frac{1}{2} QV = \frac{1}{2} \times (3.0 \times 10^{-3} \text{ C}) \times 60 \text{ V} = 0.090 \text{ J}$$

3.

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times (50 \times 10^{-6} \text{ F}) \times (120 \text{ V})^2 = 0.36 \text{ J}$$

4. If the pd is doubled, the energy stored goes up by a factor of four; energy stored is proportional to the square of the pd

5.

$$\begin{aligned} V &= \sqrt{\frac{2E}{C}} \\ &= \sqrt{\frac{2 \times 2.0 \text{ J}}{1000 \times 10^{-6} \text{ F}}} = 63 \text{ V} \end{aligned}$$

6.

$$Q = CV = (1000 \times 10^{-6} \text{ F}) \times 16 \text{ V} = 0.016 \text{ C}$$

$$E = \frac{1}{2} QV = \frac{1}{2} \times 0.016 \text{ C} \times 16 \text{ V} = 0.13 \text{ J}$$

$$V = \frac{Q}{C} = \frac{0.01 \text{ C}}{10 \times 10^{-3} \text{ F}} = 1.0 \text{ V}$$

$$E = \frac{1}{2} QV = \frac{1}{2} \times 0.01 \text{ C} \times 1.0 \text{ V} = 5 \text{ mJ}$$

$$\begin{aligned} V &= \sqrt{\frac{2E}{C}} \\ &= \sqrt{\frac{2 \times 100 \text{ J}}{1.0 \text{ F}}} = 14 \text{ V} \end{aligned}$$

$$Q = CV = 1.0 \text{ F} \times 14 \text{ V} = 14 \text{ C}$$

$$C = \frac{Q}{V}$$

$$= \frac{2.0 \times 10^{-3} \text{ C}}{5000 \text{ V}} = 4.0 \times 10^{-7} \text{ F}$$

$$E = \frac{1}{2} QV$$

$$= \frac{1}{2} \times (2.0 \times 10^{-3} \text{ C}) \times 5000 \text{ V} = 5.0 \text{ J}$$

$$Q = \frac{2E}{V}$$

$$= \frac{2 \times 50 \times 10^{-3} \text{ J}}{100 \text{ V}} = 1.0 \times 10^{-3} \text{ C}$$

$$C = \frac{Q}{V}$$

$$= \frac{1.0 \times 10^{-3} \text{ C}}{100 \text{ V}} = 10 \text{ } \mu\text{F}$$

$$V = \sqrt{\frac{2E}{C}}$$

$$= \sqrt{\frac{2 \times 2.0 \text{ J}}{33000 \times 10^{-6} \text{ F}}} = 11 \text{ V}$$

$$Q = CV = (33000 \times 10^{-6} \text{ F}) \times 11 \text{ V} = 0.36 \text{ C}$$

7.

$$Q = CV = (1.0 \times 10^{-6} \text{ F}) \times 10 \text{ V} = 1.0 \times 10^{-5} \text{ C}$$

8.

$$1.0 \times 10^{-5} \text{ C}$$

10.

$$E = \frac{1}{2} QV = \frac{1}{2} \times (1.0 \times 10^{-5} \text{ C}) \times 10 \text{ V} = 5.0 \times 10^{-5} \text{ J}$$

11.

$$E = QV = (1.0 \times 10^{-5} \text{ C}) \times 10 \text{ V} = 1.0 \times 10^{-4} \text{ J}$$

12. 50% of the energy transferred from the battery is used to heat the connecting wires.

External references

This activity is taken from Advancing Physics, Chapter 10, 110S

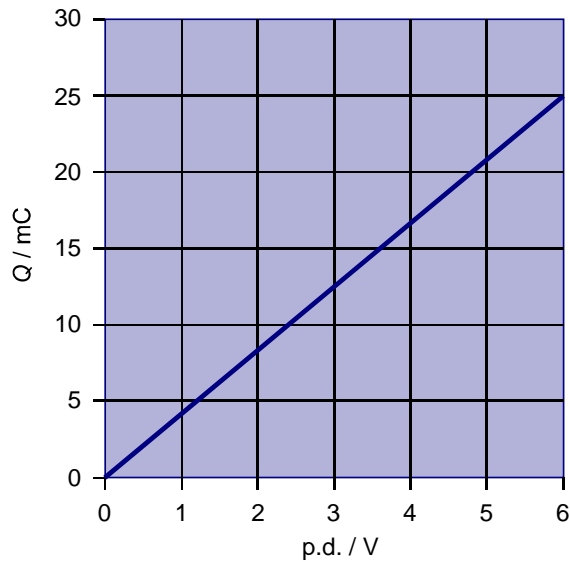
TAP 128- 5: Energy to and from capacitors

What to do

Work your way through the questions, preferably in the order in which they appear. The first few questions will give you some practice in using the equations relating to energy storage.

Questions

The graph shows the relation between the charge on a capacitor and the pd across it.



1. Use the graph to find the capacitance of the capacitor.
2. Use the graph to find the energy stored, first when the pd across the capacitor is 4.0 V and again when the pd is 6.0 V.

A capacitor in a catalogue is marked 33 000 μF , 10 V. It also has a mass of 40 g. It is charged to 10 V.

3. Calculate the energy stored.
4. If all energy stored could be converted to kinetic energy, how fast would the capacitor be moving?

5. If all the stored energy could be converted to gravitational potential energy, how high would it rise?

Practical advice

The range of questions is designed to give students practice in handling the relevant equations.

Answers and worked solutions

1.

$$\text{Capacitance} = \text{gradient} = \frac{25 \times 10^{-3} \text{ C}}{6.0 \text{ V}} = 4.2 \text{ mF.}$$

2.

$$E_4 = \text{area under the graph to } 4.0 \text{ V} = \frac{1}{2} \times 4.0 \text{ V} \times 17 \times 10^{-3} \text{ C} = 0.034 \text{ J}$$

$$E_6 = \text{area under the graph to } 6.0 \text{ V} = \frac{1}{2} \times 6.0 \text{ V} \times 25 \times 10^{-3} \text{ C} = 0.075 \text{ J}$$

3.

$$\begin{aligned} E &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} \times 33000 \times 10^{-6} \text{ F} \times (10 \text{ V})^2 \\ &= 1.65 \text{ J} \end{aligned}$$

4.

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 1.65 \text{ J}}{40 \times 10^{-3} \text{ kg}}} = 9.1 \text{ m s}^{-1}$$

5.

$$1.65 \text{ J} = (40 \times 10^{-3} \text{ kg}) \times 10 \text{ N kg}^{-1} \times h.$$

Therefore h is 4.1 m.

External references

This activity is taken from Advancing Physics Chapter 10, 120S