TAP 128- 4: Energy stored in capacitors.

What to do

Work your way through the questions, preferably in the order in which they appear. The first few questions will give you some practice in using the equations relating to energy storage.

Questions

A 50 microfarad (μF) capacitor is charged to a pd of 60 V.

1. Calculate the charge on the capacitor.

2. Calculate the energy stored.

3. Calculate the energy stored when the pd is doubled to 120 V.

4. Compare your answers to questions 2 and 3. What does this tell you about the relationship between the energy stored by a capacitor and the pd to which it has been charged?

5. A 1000 μF capacitor is charged so that its stores 2.0 J of energy. Calculate the pd to which it has been charged.

6. The incomplete table below contains values of capacitance, charge, pd and energy for a series of charged capacitors. Carry out calculations and fill in the blanks in the table.

Capacitance	**Charge**	**Potential difference**	**Energy**
1000 μF		16 V	
10 mF	0.01 C		
1.0 F			100 J
	2.0 mC	5000 V	
		100 V	50 mJ
33 000 μF			2.0 J

A 1.0 μF capacitor is charged to a pd of 10 V.

7. Calculate the charge on the capacitor.

8. How much charge flowed through the battery during charging?

9. How many electrons flowed through the battery during charging?

10. Calculate the energy stored by the capacitor.

11. How much energy was transferred from the battery during the charging process?

12. (Rather harder) You should have found different answers for questions 10 and 11. Explain this difference.

Hints

1. You will have to do quite a bit of rearranging of equations.
6. Think of the definition of potential difference when you are trying to work out how much energy is involved in passing charge through a battery.

Practical advice

The range of questions is designed to give students practice in handling the relevant equations. It is advisable that students work through the questions more or less in the order they appear. Because there are three different equations relating to energy storage in capacitors, there are several instances where students have to choose which to use for their own convenience.

There are many pitfalls, not deliberate, into which students can fall because of difficulties with unit prefixes and with standard form. From this point of view, the questions provide some good practice in handling both of these issues.

If they have met only E = ½ Q V but not the other versions, it may be necessary to spend some time with them developing ‘variations on the theme’. Question 12 is an issue familiar to teachers but difficult for most students to grasp; they might need more help than is given in the hints. It is probably best left as extension work for students aiming for higher grades.

Answers and worked solutions

1. 3.0×10^{-3} C

2. 0.090 J

3. 0.36 J

4. If the pd is doubled, energy stored goes up by factor of four; energy is proportional to the square of the pd

5. 63 V

6.

Capacitance	Charge	Potential difference	Energy
1000 μF	0.016 C	16 V	0.013 J
10 mF	0.01 C	1.0 V	5 mJ
1.0 F	14 C	14 V	100 J
4.0×10^{-7} F	2.0 mC	5000 V	5 J
10 μF	1.0mc	100 V	50 mJ
33 000 μF	0.36 C	11 V	2.0 J

7. 1.0×10^{-5} C

8. 1.0×10^{-5} C

9. 6.2×10^{13}

10. 5.0×10^{-5} J

11. 10.0×10^{-5} J

12. 50% of the energy from the battery is used to heat the connecting wires (warming the wires). If there were no resistance somewhere in the circuit we would have a non-physical situation of the system discharging in an infinitesimal time, with infinite currents. In practice inductive effects would also come into play, even if we had a superconducting circuit. This raises the issue of boundary conditions- you can’t have a non-physical system.

(See Physics Education July 2005 pages 370 and following.

For details how to obtain Physics Education see the IOP website.)

Worked solutions

1.

$$Q = CV = (50 \times 10^{-6}\ \text{F}) \times 60\ \text{V} = 3.0 \times 10^{-3}\ \text{C}$$

2.

$$E = \tfrac{1}{2}QV = \tfrac{1}{2} \times (3.0 \times 10^{-3}\ \text{C}) \times 60\ \text{V} = 0.090\ \text{J}$$

3.

$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times (50 \times 10^{-6}\ \text{F}) \times (120\ \text{V})^2 = 0.36\ \text{J}$$

4. If the pd is doubled, the energy stored goes up by a factor of four; energy stored is proportional to the square of the pd

5.

$$V = \sqrt{\frac{2E}{C}}$$

$$= \sqrt{\frac{2 \times 2.0\ \text{J}}{1000 \times 10^{-6}\ \text{F}}} = 63\ \text{V}$$

6.

$$Q = CV = (1000 \times 10^{-6}\ \text{F}) \times 16\ \text{V} = 0.016\ \text{C}$$

$$E = \tfrac{1}{2}QV = \tfrac{1}{2} \times 0.016\ \text{C} \times 16\ \text{V} = 0.13\ \text{J}$$

$$V = \frac{Q}{C} = \frac{0.01C}{10 \times 10^{-3}\ \text{F}} = 1.0\ \text{V}$$

$$E = \tfrac{1}{2}QV = \tfrac{1}{2} \times 0.01\ \text{C} \times 1.0\ \text{V} = 5\ \text{mJ}$$

$$V = \sqrt{\frac{2E}{C}}$$

$$= \sqrt{\frac{2 \times 100\ \text{J}}{1.0\ \text{F}}} = 14\ \text{V}$$

$$Q = CV = 1.0\ \text{F} \times 14\ \text{V} = 14\ \text{C}$$

$$C = \frac{Q}{V}$$

$$= \frac{2.0 \times 10^{-3}\ \text{C}}{5000\ \text{V}} = 4.0 \times 10^{-7}\ \text{F}$$

$$E = \tfrac{1}{2}QV$$

$$= \tfrac{1}{2} \times (2.0 \times 10^{-3}\ \text{C}) \times 5000\ \text{V} = 5.0\ \text{J}$$

$$Q = \frac{2E}{V}$$

$$= \frac{2 \times 50 \times 10^{-3}\ \text{J}}{100\ \text{V}} = 1.0 \times 10^{-3}\ \text{C}$$

$$C = \frac{Q}{V}$$

$$= \frac{1.0 \times 10^{-3}\ \text{C}}{100\ \text{V}} = 10\ \mu\text{F}$$

$$V = \sqrt{\frac{2E}{C}}$$

$$= \sqrt{\frac{2 \times 2.0\ \text{J}}{33\,000 \times 10^{-6}\ \text{F}}} = 11\ \text{V}$$

$$Q = CV = (33\,000 \times 10^{-6}\ \text{F}) \times 11\ \text{V} = 0.36\ \text{C}$$

7.

$$Q = CV = (1.0 \times 10^{-6}\ \text{F}) \times 10\ \text{V} = 1.0 \times 10^{-5}\ \text{C}$$

8.

$$1.0 \times 10^{-5}\ \text{C}$$

10.

$$E = \tfrac{1}{2}QV = \tfrac{1}{2} \times (1.0 \times 10^{-5}\ \text{C}) \times 10\ \text{V} = 5.0 \times 10^{-5}\ \text{J}$$

11.

$$E = QV = \left(1.0 \times 10^{-5}\ \text{C}\right) \times 10\ \text{V} = 1.0 \times 10^{-4}\ \text{J}$$

12. 50% of the energy transferred from the battery is used to heat the connecting wires.

External references

This activity is taken from Advancing Physics, Chapter 10, 110S