

## Episode 126: Capacitance and the equation $C = Q/V$

Having established that there is charge on each capacitor plate, the next stage is to establish the relationship between charge and potential difference across the capacitor.

### Summary

**Demonstration: Charging a capacitor. (10 minutes)**

**Discussion: Defining capacitance and the farad. (20 minutes)**

**Student experiment: Charge proportional to voltage – two alternatives. (30 minutes)**

**Discussion: Factors affecting C. (10 minutes)**

**Student experiment: Factors affecting C. (30 minutes)**

**Discussion: Permittivity. (20 minutes)**

**Discussion: Working with real capacitors (10 minutes)**

**Student questions + discussion: Calculations with real capacitors. (20 minutes)**

### Demonstration:

#### Charging a capacitor

The experimental demonstration 'charging a capacitor at a constant rate' shows that the potential difference across the capacitor is proportional to the charge.

TAP 126-1: Charging a capacitor at constant current

### Discussion:

#### Defining capacitance and the farad

The experiment shows that  $Q \propto V$ , or  $Q = \text{constant} \times V$ . This constant is called the capacitance,  $C$ , of the capacitor and this is measured in farads (F). So capacitance is charge stored per volt, and farads = coulombs/volts.

It is a good idea to point out that 1 farad is a very large capacitance and that most capacitors will be micro,  $\mu$ , - ( $10^{-6}$ ), nano- ( $10^{-9}$ ), or pico- ( $10^{-12}$ ) farads. The capacitance of the planet Earth, considered as an isolated sphere of radius  $R$ , using  $C = 4 \pi \epsilon_0 \epsilon_r R$  is  $710 \mu\text{F}$ .

### Student experiment:

#### Charge proportional to voltage – first alternative

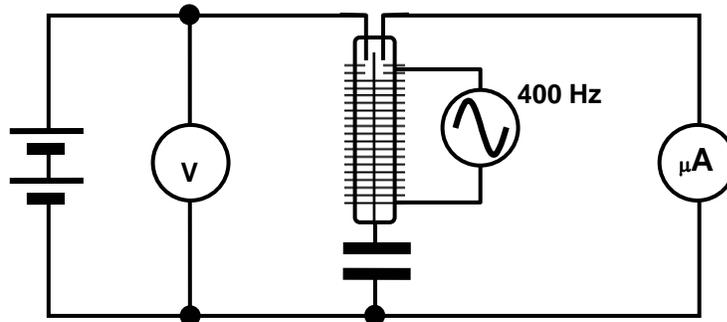
The relationship between charge and potential difference can be investigated further by the students themselves. Two experiments are possible; this one makes use of a coulomb meter.

By charging a suitable capacitor to different voltages and measuring the charge stored each time, you have a rapid confirmation of the relationship  $Q \propto V$ . The experiment can be repeated with different capacitors. Plot a graph of  $Q$  against  $V$ .

TAP 126-2: Measuring the charge on a capacitor

### Charge proportional to voltage – second alternative

The second investigation of the relationship between charge and pd makes use of a change-over



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reed switch. Students may have met simple on/off reed switches in technology or even in primary school.

Although this is a more difficult experiment to perform, it has value because it can be extended to investigate the factors determining capacitance of a parallel plate capacitor if this is needed for your specification.

From either experiment, a graph of  $Q$  against  $V$  can be plotted. This is helpful later when discussing the energy stored in a capacitor. (N.B. The graph from a reed switch experiment will not pass through the origin so the effect of stray capacitance in the experiment will have to be explained)

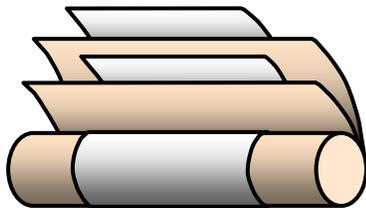
TAP 126-3: Using a reed switch to measure capacitance

### Discussion:

#### Factors affecting $C$

If your specification requires the study of the equation  $C = \epsilon_0 \epsilon_r A/d$  then this is a convenient point to cover that work.

It is a good time to introduce the idea that many 'tubular' shaped capacitors are, in fact, a parallel plate capacitor which has been rolled up and filled with a dielectric. Why? (A large area with a small gap gives reasonable values of capacitance; dielectric increases capacitance; rolling reduces the overall size.)



## **Student experiment:**

### **Factors affecting C**

Using a reed switch, or a digital capacitance meter, investigate the factors determining capacitance for a parallel plate capacitor.

If you do not have a reed switch many cheap digital multimeters now have a capacitance meter that covers the pF and nF range, which will work effectively here.

By using parallel plates as the capacitor in this experiment, the relationship between capacitance and area can be found by altering the area of overlap while using spacers leads to the relationship between capacitance and separation. Placing plastic sheets between the plates shows the effect of a dielectric and shows why the relative permittivity appears in the formula. If time is short, these three experiments could be done as group activities, with groups reporting back on their findings.

## **Discussion:**

### **Permittivity**

Discuss the outcomes of the experiments and the significance of  $\epsilon_0$ , the permittivity of free space. Deduce its units of  $\text{F m}^{-1}$  or  $\text{C}^2 \text{N}^{-1} \text{m}^{-2}$ .

## **Discussion:**

### **Working with real capacitors**

Take a selection of capacitors and look at the information written on each. This will include the capacitance and the maximum working voltage. On an electrolytic capacitor there will also be an indication of the polarity for each terminal (and there may be a maximum ripple current).

Discuss what the markings mean and compare the charge stored by each capacitor at maximum voltage (practice in using  $Q = C \times V$ ).

How does this relate to the physical size of the capacitor? (This is unlikely to be simply that the larger the capacitance the bigger the capacitor. The working voltage is important, as is the material between the plates.)

## **Student questions:**

### **Calculations with real capacitors**

Follow-up questions will round off this episode.

TAP 126-4: Charging capacitors questions

TAP 126-5: Problems on capacitors

## TAP 126- 1: Charging a capacitor at constant current

### Background to the experiment

When a capacitor is charged by connecting it to a battery or other dc power supply, the current in the circuit gradually falls to zero. The rate at which this happens depends on both the capacitance of the capacitor and the presence of any resistance in the circuit. If the resistance of the circuit is high, the current will be correspondingly small and the capacitor will charge up more slowly than if there were less resistance in the circuit.

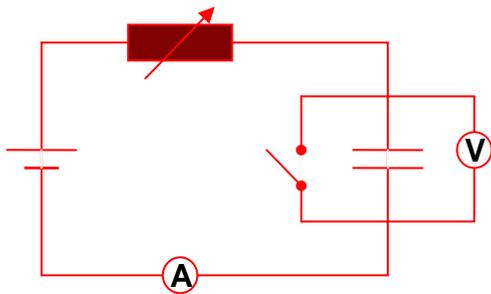
Using a variable resistor, with a bit of manual dexterity, you can keep the current constant and time how long it takes to charge a capacitor. By knowing both the time and the current, you can determine the charge stored on the capacitor.

Then, by charging the capacitor to different voltages, you can establish experimentally the relationship between the amount of charge and the pd across the capacitor resulting from it.

### You will need

- ✓ a partner
- ✓ power supply, 5 V dc
- ✓ digital multimeter, used as ammeter
- ✓ digital multimeter, used as voltmeter
- ✓ capacitor, 470  $\mu\text{F}$
- ✓ potentiometer, mounted with 4 mm sockets, 100  $\text{k}\Omega$
- ✓ leads, 4 mm plus shorting switch
- ✓ hand-held stop watch

### Constant current charging



Watch the supply pump a fixed number of coulombs onto each capacitor plate each second.

How is the charge on one plate changing with time? So how is the pd changing with time? Can you think of a way to show this?

## Examining the results

From the current and time measurements, you can use  $Q = It$  to determine the amount of charge which flowed onto the capacitor plates. If you do this for cunningly chosen measurements, you are now in a position to examine the relationship between the charge and the pd across the capacitor which results from the redistribution of charge on its surfaces. A quick look at the results will show that more charge is needed to raise the pd to a greater value.

## Outcomes

1. You will understand better the process by which a capacitor is charged.
2. By focusing on the simplicity of charging at a constant current, you can see how the redistribution of charge results in a potential difference across the capacitor.
3. You will be able to see that this leads to the relationship 'pd across a capacitor is proportional to the charge stored on one plate of the capacitor'.

## Practical advice

Before showing students this demonstration, they should be aware of current as a flow of charge in a circuit and have tackled some of the problems involving the calculation of charge and the use of  $Q = It$ . They might be shown initially what happens when the capacitor is charged without changing the resistance, i.e. that the current gradually falls, and this should simply be introduced as a nuisance at this stage. They need to be clear that the variable resistance is there simply to allow you to keep the current constant. No other explanation is necessary at this stage.

Close the switch and use the variable resistor to set the ammeter to some convenient value, e.g. 100 mA. When you are ready, remove the shorting link across the capacitor, start timing and adjust the control on the variable resistor to maintain the current at the value you have set. You will probably have to practise this a few times – it can be quite tricky. Carry on adjusting the resistor for as long as you can. Connect the digital voltmeter briefly across the capacitor to measure the pd that has been generated by the redistribution of charge.

Record the charging current and the time as well as the pd across the capacitor. You will probably wish to repeat this measurement at least once or twice to allow for the difficulty you have in keeping the current constant.

You might repeat this process using different pds, if you want to take it further, or to use a computer-based oscilloscope to get plots of charging current / time and pd across capacitor / time.

You will probably not have time to do the demo in great detail, but it is well worth the effort to practise a bit at keeping the current steady – surprisingly difficult if you have not done it before.

Explain the function of the short circuit link across the capacitor, which allows you to set the initial charging current to a convenient value. You will have already taught that the capacitor is an unusual device, which does not permit a flow of current through it. You will also have to explain away that you eventually run out of control when you have reduced the resistance to zero and the current will then fall. A simple and effective, but not totally accurate procedure is to keep on timing until the current has fallen to half the initial value. For an appropriate group of able students, this could be an additional teaching point.

If you have done the experiment reasonably carefully, and have results for different pds, students can plot a graph of Q against pd

If you have time, it is a good idea to let students have a try as well.

### **Alternative approaches**

The approach will depend on students' prior knowledge. If they already know  $Q = CV$ , then the activity becomes one in showing that the different gradients of the graph relate to different values of the capacitance. You may wish to direct students to use it from this standpoint, particularly if there is time for students to do the experiment themselves.

### **External references**

This activity is taken from Advancing Physics Chapter 10, 120D

## TAP 126- 2: Measuring the charge on a capacitor

See also TAP 126-3

### Relating quantities

You will use a coulomb meter to measure the quantity of charge stored on a capacitor and to investigate how this varies with the applied voltage. This leads to measurements of capacitance.

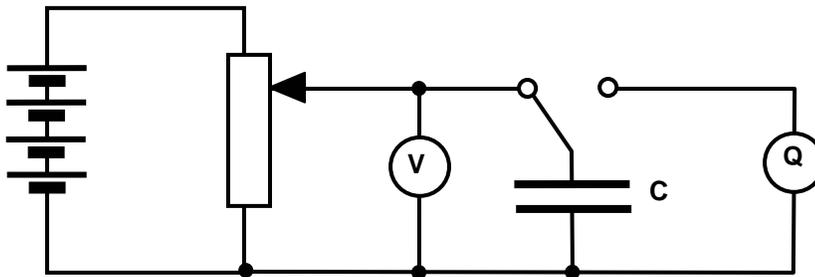
### You will need

- ✓ digital coulomb meter
- ✓ digital multimeter
- ✓ capacitors  $0.1\ \mu\text{F}$ ,  $0.22\ \mu\text{F}$ ,  $0.047\ \mu\text{F}$
- ✓ clip component holder
- ✓ potentiometer,  $1\ \text{k}\Omega$
- ✓ spdt switch
- ✓ power supply,  $5\ \text{V}$  dc or  $6\ \text{V}$  battery pack
- ✓ leads,  $4\ \text{mm}$

### The activity

In this experiment a voltage supply is connected across the terminals of a capacitor and the resulting charge stored by the capacitor is measured directly using a coulomb meter.

Choose a capacitor of  $0.1\ \mu\text{F}$ , insert it in the component holder and assemble this circuit.



Adjust the voltage output from the potentiometer to  $0.5\ \text{V}$ . Place the switch into the left-hand position. This applies the voltage across the terminals of the capacitor with the result that it

becomes charged. The amount of charge on the capacitor is shown by the coulomb meter when the switch is moved to the right-hand position. Record the values of voltage and charge and enter them in the table below.

Repeat the procedure to obtain a series of readings for the voltages shown in the table.

	Capacitor 1	Capacitor 2	Capacitor 3
pd / V	Charge / nC	Charge / nC	Charge / nC
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			
3.5			
4.0			
4.5			
5.0			

Repeat the experiment with different values of capacitance.

### Analysing the results

Plot the readings for charge against voltage on common axes for the three capacitors.

Do the shapes of your graphs support the idea that the charge stored varies in proportion to the voltage applied? Explain your reasoning.

Calculate the gradient of each graph. The value obtained is a measure of the value of the capacitance in each case. Compare this with the value marked on the side of the capacitor. (It may seem rather odd measuring something which you apparently know already, but individual capacitors are notorious for having capacitance values slightly different from that printed on their outside. The marked value is only a rough guide. This is particularly so for electrolytic capacitors which often differ by more than 10%. The experimental method described here generally gives a much more precise measure of the value of an individual capacitor.)

### Thinking about the theory

Since capacitance is defined as  $C = Q / V$ , we can write  $Q = CV$ . Thus, for a graph of  $Q$  against  $V$  the gradient is equal to  $C$ .

Comparing the graphs of  $Q$  against  $V$  with the different capacitors in your experiment, which graph stores the most charge at a given voltage? Does this correspond to the largest value capacitor? Explain your reasoning.

### **Things to remember**

1. The graphs show that the charge stored on a capacitor varies in direct proportion to the voltage applied.
2. The value of a capacitor can be calculated from the gradient of a Q versus V graph.
3. When comparing capacitors, the magnitude of the charge stored on a capacitor at a given voltage is larger for a larger value capacitor.

## **Practical advice**

The coulomb meter is a useful instrument for measuring the charge stored on small-value capacitors. A typical coulomb meter can measure up to 2 mC. For voltages of up to 6 V this implies that it can be used with capacitors up to a maximum value of 0.3 mF.

The use of the coulomb meter described here enables students to explore the proportionality between  $Q$  and  $V$  fairly directly with the minimum of calculation. The measurements are of a generally better precision than those associated with the transient currents involved with discharging capacitors.

## **Alternative approaches**

If time is short, this could be done as a demonstration with little modification. The experiment could be conducted with the capacitor markings hidden, so that the objective could be more sharply focused on obtaining a measurement of the capacitor value. A spreadsheet could be used for recording the results and evaluating the gradients of the graphs.

## **External references**

This activity is taken from Advancing Physics Chapter 10, 110E

## **Student experiment – second alternative**

TAP 126-3: Using a reed switch to measure capacitance

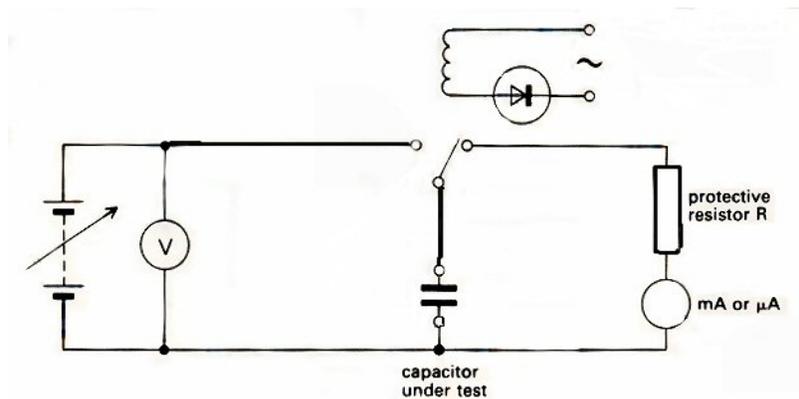
## TAP 126- 3: Using a reed switch to measure capacitance

### Relating quantities

You will use a reed switch with a capacitor to investigate how charge varies with the applied voltage. This leads to measurements of capacitance.

### You will need

- ✓ reed switch (change-over type in coil)
- ✓ signal generator
- ✓ low voltage power supply and smoothing unit dc (up to 20 V), or twelve 1.5 V cells. If commercial smoothing units are not available a high-ripple electrolytic capacitor (in parallel with the supply output) will do.
- ✓ 1 pair capacitor plates 25 cm x 25 cm with sixteen 1cmx1cm polythene spacers about 1mm thickness
- ✓ 100 k $\Omega$  protective resistor
- ✓ clip component holders
- ✓ milli or micro ammeter.
- ✓ Voltmeter that reads to 20 V
- ✓ 4 mm leads



### The activity

With a diode rectifying the ac from the signal generator the reed switch charges and discharges once per cycle. The capacitor discharge current is measured.

Charge per second =  $I = \text{current} = Q f$  where  $f$  is the frequency of the signal generator.

$$C = Q / V = I / f V$$

Find  $I$  and  $V$  from the meter readings.

Vary the voltage and measure the current. Record the results and calculate the charge.

Drawing a graph of charge against voltage enables the capacitance to be found.

### **Practical advice**

Frequency should be about 400 Hz and the protective resistance about 100 k $\Omega$ . The capacitor plates must not touch whilst the reed switch is working nor should the switch operating pd be exceeded. The signal generator pd should be raised until the switch is heard vibrating.

If a stock capacitor is used rather than the plates then a 220  $\Omega$  protective resistor is required for capacitances of a few microfarads and a milliammeter will be required

To protect the reed switch, the capacitor plates should never touch whilst the switch is working. Switch off and disconnect the power supply before adjusting the plates. For accuracy, keep hands and earthed objects well away from the plates whilst taking readings.

### **Extension**

The can be extended to investigate the factors determining capacitance of a parallel plate capacitor if this is needed for your specification.

(N.B. The graph from a reed switch experiment will not pass through the origin so the effect of stray capacitance in the experiment will have to be explained. This is particularly important if you vary the area of the plates. Suggested areas are  $3/4$ ,  $2/3$ ,  $1/2$ ,  $1/3$ . A weight will be needed to counterbalance the part of the plate sticking out. This should be over the edge of a bench to try to reduce stray capacitance with the bench.)

If the plates are used the effect of different materials between the plates can be demonstrated, e.g. plastic sheet, cardboard, rubber, a newspaper for example as well as increasing the number of spacers and so increasing the gap

If fixed capacitors are used then the effect of adding capacitors in series and parallel can be demonstrated. Be aware that the marked value on a capacitor may be correct only to within say 50%.

### **External references**

This activity is taken from an adaptation of Revised Nuffield Advanced Physics experiment E7.

## TAP 126- 4: Charging capacitors

### What to do

Answer the questions. They develop a line of thought, so answering them in order is likely to help.

### Questions

In an experiment a capacitor is charged from a constant current supply by a 100 mA current pulse which lasts 25 s.

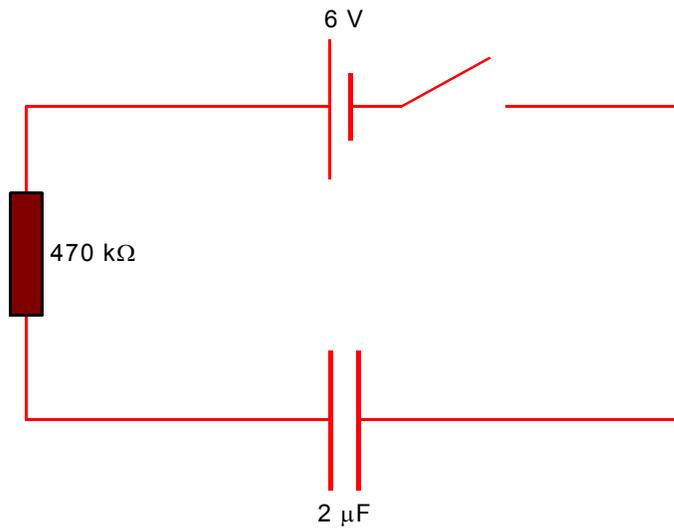
1. Calculate the charge on the capacitor after this time.
2. The pd across the capacitor is 6 V when it has been charged. Calculate the capacitance of the capacitor.

A capacitor is charged at a constant current of 2.0 mA until the charge on the capacitor is 0.010 C.

3. How long did it take the capacitor to charge to this value?
4. The capacitance of the capacitor is 100 microfarad ( $\mu\text{F}$ ). To what pd had it been charged?
5. This question allows you to practise using the equation  $Q = C V$ . Fill in the gaps in the table.

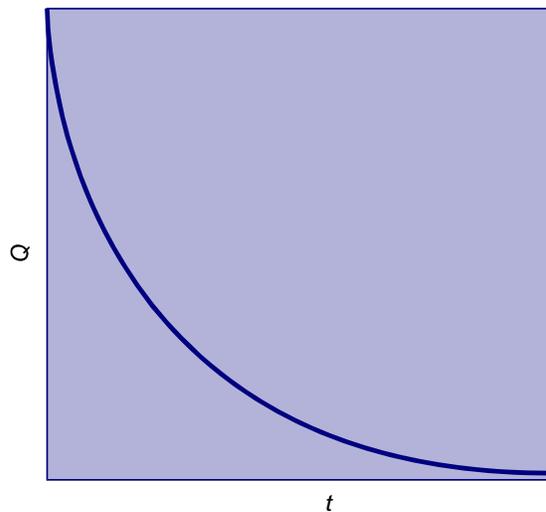
C	Q	V
1000 $\mu\text{F}$		12 V
	2 mC	250 V
32 $\mu\text{F}$	0.64 mC	
33 mF		6 V
	1.25 C	25 kV
10 mF	2.5 C	

In this circuit the capacitor is initially uncharged.



6. Calculate the current through the resistor when the switch is first closed.
  
7. What is the current after the switch has been closed for a long time? Explain your answer.
  
8. Calculate the current through the resistor when the pd across the capacitor is  $2.0\text{ V}$ .

When this capacitor is charged through the fixed resistor, the graph shows how the charge on the capacitor varies with time.



9. What is the significance of the gradient of the graph?
10. Explain why the gradient of the graph gradually falls?
11. Calculate the maximum charge on the capacitor.
12. Use the same axes to sketch the curve you would expect if the resistor were replaced by one with a smaller resistance.

### Practical advice

Students are expected to have met the equation  $Q = CV$  and to understand the definition of capacitance as the ratio of charge to potential difference. They should also be familiar with the farad as the unit of capacitance but also the submultiples of 'milli' and 'micro' as more common variants since the farad is such a large unit.

The first questions refer to a capacitor having being charged at constant current and it will be useful if they have either seen this demonstrated or, if possible, done the experiment themselves. It offers a good way to see how charge accumulates on a capacitor with the corresponding build-up of pd across it.

Question 5 is designed to give practice in using  $Q = CV$  and, in particular to sort out their 'millis' from their 'micros'.

The third group of questions concentrates on charging a capacitor through a fixed resistance with the corresponding fall in current as the pd across the capacitor rises. They must relate the gradient of the graph to the current flowing. In your teaching, you may wish to direct some students towards the differential equation which describes the curve mathematically, but this is not appropriate for all.

### Solutions and worked examples

1. 2.5 C
2. 0.42 F
3. 5 s
4. 100 V
- 5.

C	Q	V
1000 $\mu$ F	0.012C	12 V
8 $\mu$ F	2 mC	250 V
32 $\mu$ F	0.64 mC	20V
33 mF	0.02C	6 V
50 $\mu$ F	1.25 C	25 kV
10 mF	2.5 C	250V

6. 13 mA
7. Zero
8. 8.5 mA
9. The gradient is the current at that time.
10. As the capacitor charges up, the pd across it rises and the pd across the resistor falls. The current through the resistor is equal to the pd across it divided by the resistance, so the current, and therefore the gradient, gradually decreases.
11. 12 mC
12. The graph is steeper; it reaches maximum charge in a shorter time.

### Worked solutions

1.  $Q = It = 100 \times 10^{-3} \text{ A} \times 25 \text{ s} = 2.5 \text{ C}$

2.

$$C = \frac{Q}{V} = \frac{2.5 \text{ C}}{6.0 \text{ V}} = 0.42 \text{ F}$$

3.

$$t = \frac{Q}{I} = \frac{0.010 \text{ C}}{2.0 \times 10^{-3} \text{ A}} = 5 \text{ s}$$

4.

$$V = \frac{Q}{C} = \frac{0.010 \text{ C}}{100 \times 10^{-6} \text{ F}} = 100 \text{ V}$$

5.

$$Q = CV = 1000 \times 10^{-6} \text{ F} \times 12 \text{ V} = 0.012 \text{ C}$$

$$C = \frac{Q}{V} = \frac{2 \times 10^{-3} \text{ C}}{250 \text{ V}} = 8 \times 10^{-6} \text{ F}$$

$$V = \frac{Q}{C} = \frac{0.64 \times 10^{-3} \text{ C}}{32 \times 10^{-6} \text{ F}} = 20 \text{ V}$$

$$Q = CV = 33 \times 10^{-3} \text{ C} \times 6.0 \text{ V} = 0.20 \text{ C}$$

$$C = \frac{Q}{V} = \frac{1.25 \text{ C}}{25 \times 10^3 \text{ V}} = 50 \times 10^{-6} \text{ F}$$

$$V = \frac{Q}{C} = \frac{2.5 \text{ C}}{10 \times 10^{-3} \text{ F}} = 250 \text{ V}$$

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{470 \times 10^3 \Omega} = 1.3 \times 10^{-5} \text{ A}$$

6.

7. Zero, because when capacitor is fully charged, the pd across the resistor is zero and so no current flows through it.

8. The pd across the resistor is 4.0 V, therefore

$$I = \frac{4.0 \text{ V}}{470 \times 10^3 \Omega} = 8.5 \times 10^{-6} \text{ A.}$$

9. The gradient is the current at that time.

10. As the capacitor charges up, the pd across it rises and the pd across the resistor falls. The current through the resistor is equal to the pd across it divided by the resistance, so the current, and therefore the gradient, gradually decreases.

11.

$$Q = CV = 2 \times 10^{-6} \text{ F} \times 6.0 \text{ V} = 12 \times 10^{-6} \text{ C}$$

12. With smaller resistance the initial current is greater so the graph is initially steeper. The pd across the capacitor rises more quickly and so the capacitor takes less time to charge.

## **External references**

This activity is taken from Advancing Physics Chapter 10, 60S

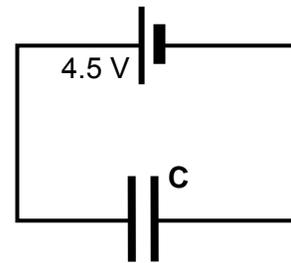
## TAP126- 5: Problems on capacitors

Electron charge =  $-1.6 \times 10^{-19}$  C

1. What is meant by the capacitance of a capacitor?
2. Define the farad.
3. (a) A capacitor of capacitance  $5 \mu\text{F}$  is connected to a 6 V supply. What charge is stored in the capacitor?  
(b) A  $400 \text{ pF}$  capacitor carries a charge of  $2.5 \times 10^{-8}$  C. What is the potential difference across the plates of the capacitor?
4. A capacitor is charged such that there is a charge of  $+20 \text{ mC}$  on the positive plate. What is the charge on the negative plate?

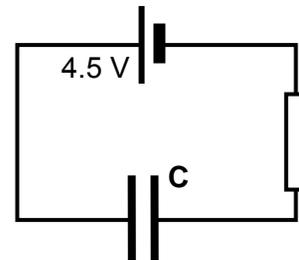
5. A  $4700 \mu\text{F}$  capacitor is connected as shown in the circuit diagram. When it is fully charged:

- (a) what is the charge on the positive plate of the capacitor?
- (b) what is the potential difference across the capacitor?
- (c) how many additional electrons are on the negative plate?



6. A resistor of  $100 \Omega$  is now added to the circuit as shown in the second diagram.

- (a) What effect does this have on the time to charge up the capacitor?
- (b) What is the final charge on the plates?
- (c) What is the final potential difference across the capacitor?



## Solution and worked examples

1. The ratio of the charge on the plates of the capacitor to the potential difference across them.

$$C = Q/V.$$

2. The charge required to increase the potential difference across the plates by 1 V.

3. Use  $C = Q/V$

(a)  $Q = CV = 5 \times 10^{-6} \times 6 = 30 \times 10^{-6} = 30 \mu\text{C}$

(b)  $V = Q/C = 2.5 \times 10^{-8} / 400 \times 10^{-12} = 62.5 \text{ V}$

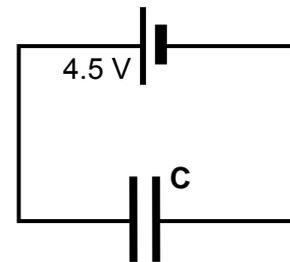
4. - 20 mC equal and opposite.

- 5.

(a)  $Q = CV = 4700 \times 10^{-6} \times 4.5 = 0.021 \text{ C}$

(b) 4.5 V

(c)  $0.021 / 1.6 \times 10^{-19} = 1.32 \times 10^{17}$

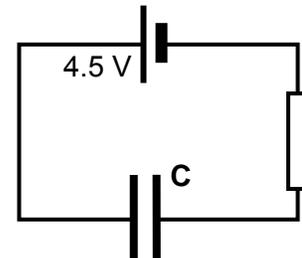


- 6.

(a) it will take longer

(b) it will be the same as before

(c) 4.5 V since no current is flowing finally there will be no potential difference across the resistor whatever its value.



## External references

This activity is taken from Resourceful Physics <http://resourcefulphysics.org/>