TAP 525-3: Fusion in a kettle?

A change of scale

When you are confident with basic calculations of fission and fusion energy changes, you should work through these questions that try to put the energies of these changes into a more human scale for you. You will also need to understand the conversion of atomic mass units to energy and the meaning of the term ‘electron volt’.

Try these

One of the reactions that fuel the stars is the fusion of two protons to give deuterium. In turn the deuterium goes through a series of reactions, the end product being helium. This is also a process that releases energy. In this question you are asked to consider the energy that would be released if all the deuterium in the water contained in an electric kettle were to be converted by fusion into helium.

The kettle contains 1 litre of water. The data you need are listed below.

1 atomic mass unit (u) = 931 MeV

1 eV = 1.6×10^{-19} J

$N_A = 6.02 \times 10^{23}$ mol^{-1}

Particle	Mass / u
${}^{1}_{1}H$	1.007 825
${}^{2}_{1}H$	2.014 102
${}^{3}_{2}He$	3.016 030
${}^{1}_{0}n$	1.008 665

1 Two deuterium nuclei ${}^{2}_{1}H$ can fuse to give one nucleus of helium ${}^{3}_{2}He$ with the ejection of one other particle. Write down the balanced equation that represents this reaction.

2 Calculate the mass change that occurs in this reaction.

3 Convert this energy into joules.

This gives you the energy released when two deuterium nuclei fuse. The next steps take you through the calculation of the total energy released if all the deuterium in the kettle water were to fuse to make helium-3. The ratio of deuterium atoms to hydrogen in water is roughly 1 to 7000.

4 What is the mass of 1 mole of water (H = 1 u; O = 16 u roughly)?

5 How many moles of water are contained in the litre?

6 How many molecules of water (H_2O) are in the kettle?

7 How many molecules of deuterium oxide (D_2O) are in the kettle?

8 Each heavy water molecule has two atoms of deuterium; what total energy is released if all the deuterium in the kettle is converted to helium-3?

Now to put this number in a new perspective. It requires 4200 J to increase the temperature of 1kg of water by 1K.

9 How many litres of water could be heated through 100 K by the fusion energy you calculated in question 8?

Hints

1 It is important to consider the atomic electrons in this equation. You begin with two, one for each hydrogen. How many electrons does an un-ionised atom of deuterium have? So what must one of the emitted particles be? This should lead you to the other particle.

2 The conversions you need are near the data table in the question.

4 The formula of water shows that there are two hydrogen atoms and one oxygen for each water molecule.

5 1 litre of water has a mass of 1 kg.

6 1 mole contains 6×10^{23} molecules of water.

Practical advice

These questions can be modified in many ways, not least by changing the homely example of a kettle to perhaps a bath full of water or even to Lake Windermere or the local reservoir.

Social and human context

The 6000 litres of heated water may not seem so significant until you realise that this has come from the fusion of deuterium which had an original volume of 0.15 cm^3.

Answers and worked solutions

1.

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$

2. Δm = (3.016 030 u + 1.008 665 u) – 2 × 2.014 102 u = – 0.0035 u

3. 0.003509 u × 931 × 10^6 eV u^{-1} × 1.6 × 10^{-19} J eV^{-1} = 5.23 × 10^{-13} J

4. 18 g

5. 1 litre of water has a mass of 1 kg.

   number of moles = 1000 g / 18 $gmol^{-1}$ = 56 mol

6. 56 mol × 6.02 × 10^{23} mol^{-1} = 3 × 10^{25}

7. (3.4 ×10^{25})/7000= 4.9 × 10^{21}

8. energy released = 4.9 ×10^{21} × (5.23 ×10^{-13} J) = 2.49 × 10^9 J

9. (2.49 ×10^9 J)/ (4200 J $kg^{-1}K^{-1}$ × 100K) = 6000 kg = 6000 litres

External reference

This activity is taken from Advancing Physics chapter 18, 260S