# TAP 525-5: Binding energy of nuclei

## Looking for patterns

You will use the data in a spreadsheet to calculate the binding energy of a set of nuclei. You will then produce a plot to show how the binding energy per nucleon varies with mass of the nucleus.

## You will need

- ✓ computer running a spreadsheet
- ✓ data provided in spreadsheet format

## Building the spreadsheet

Take a look at the four columns in the spreadsheet data. The first is simply the name of some of the stable elements. This is followed by a column showing the atomic number (*Z*, the number of protons in the nucleus) and a column giving the mass number (A, the total number of nucleons, i.e. protons plus neutrons). Finally there is a column giving the actual atomic mass. The units of this column are atomic mass units, which are defined as exactly one-twelfth of the mass of a carbon-12 atom. The atomic mass unit (u) is also called the unified atomic mass constant, and has a value of $1.660\ 5402 \times 10^{-27}$ kg.

Use this information to calculate the binding energy of each nucleus. The binding energy is simply the difference in energy between a nucleus and its constituent parts. This energy change can be measured as a change in the mass of the nucleus. A useful shortcut is that a mass difference of 1 atomic mass unit is equivalent to 931 MeV (million electron volts) of energy.

To find the binding energy you will need to subtract the mass of the constituents from the atomic mass. The constituents are Z protons, (A – Z) neutrons and Z electrons (electrons are included in the atomic mass). The masses of these in atomic mass units are:

- mass of neutron = 1.008 665 u
- mass of proton = 1.007 277 u
- mass of electron = 0.000 548 u

Create new columns in the spreadsheet giving the number of neutrons and the mass of the constituents. Now calculate the binding energy of the entire nucleus and the binding energy per nucleon. Plot this last quantity against mass number (not atomic number).

Double click on the chart below, you will need a computer running Excel.

| Element | Z | A | Mass / u |
|---|---|---|---|
| H | 1 | 1 | 1.00783 |
| He | 2 | 4 | 4.00260 |
| Li | 3 | 7 | 7.01600 |
| Be | 4 | 9 | 9.01218 |
| Be | 5 | 11 | 11.00931 |
| C | 6 | 12 | 12.00000 |
| N | 7 | 14 | 14.00307 |
| O | 8 | 16 | 15.99491 |
| F | 9 | 19 | 18.99840 |
| Ne | 10 | 20 | 19.99244 |
| Na | 11 | 23 | 22.98980 |
| Mg | 12 | 24 | 23.98504 |
| Al | 13 | 27 | 26.98153 |
| Si | 14 | 28 | 27.97693 |
| P | 15 | 31 | 30.97376 |
| Si | 16 | 32 | 31.97207 |
| Cl | 17 | 35 | 34.96885 |
| Ar | 18 | 38 | 37.96272 |
| K | 19 | 39 | 38.96371 |
| Ca | 20 | 40 | 39.96259 |
| Sc | 21 | 45 | 44.95592 |
| Ti | 22 | 47 | 46.95180 |
| V | 23 | 51 | 50.94400 |
| Cr | 24 | 52 | 51.94050 |
| Mn | 25 | 55 | 54.93810 |
| Fe | 26 | 56 | 55.93490 |
| Ni | 28 | 58 | 57.93530 |
| Co | 27 | 59 | 58.93320 |
| Cu | 29 | 63 | 62.92980 |
| Zn | 30 | 64 | 63.92910 |
| Ga | 31 | 69 | 68.92570 |
| Ge | 32 | 74 | 73.92190 |
| As | 33 | 75 | 74.92160 |
| Br | 35 | 79 | 78.91830 |
| Se | 34 | 80 | 79.91650 |
| Kr | 36 | 82 | 81.91350 |
| Rb | 37 | 85 | 84.91170 |
| Sr | 38 | 88 | 87.90560 |
| Y | 39 | 89 | 88.90540 |
| Zr | 40 | 90 | 89.90430 |
| Nb | 41 | 93 | 92.90600 |
| Mo | 42 | 98 | 97.90550 |
| Ru | 44 | 102 | 101.90370 |
| Rh | 45 | 103 | 102.90480 |
| Pd | 46 | 106 | 105.90320 |
| Ag | 47 | 107 | 106.90509 |
| Cd | 48 | 114 | 113.90360 |
| In | 49 | 115 | 114.90410 |
| Sn | 50 | 118 | 117.90180 |
| Sb | 51 | 121 | 120.90380 |
| In | 53 | 127 | 126.90040 |
| Te | 52 | 130 | 129.90670 |
| Xe | 54 | 132 | 131.90420 |
| Cs | 55 | 133 | 132.90510 |
| Ba | 56 | 138 | 137.90500 |
| La | 57 | 139 | 138.90610 |
| Ce | 58 | 140 | 139.90530 |
| Pr | 59 | 141 | 140.90740 |
| Nd | 60 | 142 | 141.90750 |
| Sm | 62 | 152 | 151.91950 |

| particle | mass /u |
|---|---|
| neutron | 1.008 665 |
| proton | 1.007 277 |
| electron | 0.000 548 |

There are four columns in the spreadsheet data.

- The name of some of the stable elements.
- The atomic number (Z, the number of protons in the nucleus).
- The mass number (A, the total number of nucleons: protons plus neutrons).
- The actual atomic mass. The units of this column are atomic mass units, which are defined as exactly one-twelfth of the mass of a carbon-12 atom. The atomic mass unit is also called the unified atomic mass constant, and has a value of $1.660\ 5402 \times 10^{-27}$ kg.

## You will have

1. A spreadsheet giving the binding energy of a selection of nuclei.
2. A graph of binding energy per nucleon against mass number.

## Practical advice

Only a selection of stable nuclei have been included, and the data have been pre-sorted so they are in mass number order rather than atomic number order, and should therefore produce a graph very readily. Students need to be encouraged to change the default settings in their spreadsheet to make the graph clearer and more easily read - an example from Excel is included here. There are some obvious spikes in the graph, which students should be encouraged to think about.

This chart is a springboard for discussing why binding energies are negative, why fission and fusion release energy and why certain nuclei are more stable than others. The chart given here indicates some of the key features.

## Alternative approaches

Use the chart given and ask students to investigate different parts of it - the long slow slope showing where fission releases energy, the steeper slope where fusion releases energy and the spikes at $^{4}$He, $^{12}$C and $^{16}$O. These are particularly important for stellar fusion.

## Social and human context

It has often been claimed that our Universe is a fluke because the values of certain fundamental constants are closely tuned to values that produce a Universe we can live in. One of these claims is that the fusion of helium in stars to produce carbon and hence all the heavier elements of which we are made requires a lucky coincidence of energy levels between $^{4}$He, $^{8}$Be (which is unstable and forms for a short time) and $^{12}$C. However, a glance at the chart shows that elements such as $^{12}$C and $^{16}$O are very close to being clusters of helium nuclei so it is, perhaps, no surprise that the relevant energy levels are close to coincidence. A good reference on this, and other aspects of basic laws, is:

Dreams of a Final Theory by Steven Weinberg (published by Vintage).

## External reference

This activity is taken from Advancing Physics chapter 18, 140s