Episode 525: Binding energy

Summary

Discussion: Introducing mass defect and atomic mass units. (10 minutes)

Discussion: Mass defect and binding energy (10 minutes)

Worked example: Calculating binding energy. (10 minutes)

Student questions: Calculations. (20 minutes)

Student activity: Spreadsheet calculations. (20 minutes)

Student activity: Spreadsheet calculations of binding energy per nucleon. (20 minutes)

Discussion: Fission and fusion linked to binding energy graph. (10 minutes)

Discussion:

Introducing mass defect and atomic mass units

Ask your students to consider whether the following data is self-consistent:

proton mass, $m_p = 1.673 \times 10^{-27}$ kg

neutron mass, $m_n = 1.675 \times 10^{-27}$ kg

mass of a ${}^{4}_{2}He$ nucleus = 6.643×10^{-27} kg

The mass of a ${}^{4}_{2}He$ nucleus is **less** than the sum of the masses of its parts; this is true for **all** nuclides. So much for conservation of mass.

Introduce the atomic mass unit (amu, or u) as a convenient unit of nuclear mass. 1 amu or 1 u = 1/12 the mass of a neutral ^{12}C atom (i.e. including its six electrons) = 1.66056×10^{-27} kg. Thus:

m_p = 1.0073 u

m_n = 1.0087 u

m_e = 0.00055 u

mass of a neutral ${}^{4}_{2}He$ atom = 4.0026 u

Discussion:

Mass defect and binding energy

What has happened to the missing mass – or mass defect – between the whole and the sum of the parts? To separate the particles, they must be pulled apart against the attractive strong force. They thus have potential energy when they are separated.

When the particles come together to form a nucleus, their potential energy decreases.

So energy must be put in to separate the nucleons of a nucleus. This energy is known as the binding energy, a rather confusing term because students often think that this means that energy is required to bind nucleons together. As with chemical bonds, this is the opposite of the truth. Energy is needed to break bonds.

Einstein's Special Theory of Relativity (1905) relates mass and energy via the equation $E = mc^2$ (where c is the speed of light in a vacuum). In this case, we have:

binding energy = mass defect $\times c^2$ or $\Delta E = \Delta m \times c^2$

(It is not advisable to talk about mass being 'converted to energy' or similar expressions. It is better to say that, in measuring an object's mass, we are determining its energy. A helium nucleus has less mass than its constituent nucleons; in pulling them apart, we do work and so give them energy; hence their mass is greater.)

Worked example:

Calculating binding energy

Calculate the mass defect and binding energy for ${}_{2}^{4}He$. (Mass defect = 0.053×10^{-27} kg; binding energy = 1.59×10^{-12} J = 9.94 MeV)

Student Questions

TAP 525-1: Change in energy: Change in mass

TAP 525-2: Finding binding energy

TAP 525-3: Fusion in a kettle?

Student activity

A data analysis exercise using Excel. This uses a spreadsheet to calculate binding energy for a number of nuclides.

TAP 525-4: A binding energy calculator

Student activity

Another spreadsheet activity, this time looking at the binding energy per nucleon. Note that it is desirable to plot this graph with a negative energy axis; this means that the lowest values are for the most stable nuclides.

TAP 525-5: Binding energy of nuclei

Discussion

Briefly discuss fission and fusion in terms of the graph. Although the fission 'jump' looks quite small compared to a typical fusion jump, the graph is plotting BE *per nucleon*. Many more nucleons are involved in the fission of heavy atoms than in the fusion of lighter ones. (This topic can be developed further when discussing nuclear power.)

TAP 525-6: Binding energy per nucleon

TAP 525-1: Change in energy: Change in mass

Calculating using $E_{rest} = mc^2$

These questions show you how to calculate changes in energy from changes in mass, using Einstein's relation $E_{rest} = mc^2$ linking the rest energy of a particle to its mass.

Transmutation of chemical elements

The dream of the ancients was alchemy: turning base metals into gold. Although this is chemically impossible, at the end of the nineteenth century radioactivity was discovered by Henri Becquerel. When alpha and beta radiation are emitted atomic nuclei are 'transmuted' from one element to another. For example:

$$^{238}_{92}\text{uranium} \rightarrow {}^{4}_{2}\,\text{alpha} + {}^{234}_{90}\text{thorium}.$$

In 1932 using protons (hydrogen nuclei) accelerated through a potential difference of 800 000 V, two English physicists, Cockcroft and Walton, carried out the first artificial transmutation: by bombarding lithium with the protons they produced two helium nuclei:

$$^{1}_{1}\text{H} + {}^{7}_{3}\text{Li} \rightarrow {}^{4}_{2}\,\text{He} + {}^{4}_{2}\text{He}.$$

Change in mass

Notice that in both these reactions the mass number and charge (proton number) are conserved. Energy, however, is only conserved if you take account of changes to the rest energy – in effect of changes to the masses – of the particles.

In Cockcroft and Walton's experiment, the masses of the particles are:

- H: 1.0073 atomic mass units
- Li: 7.0160 atomic mass units
- He: 4.0015 atomic mass units.

An atomic mass unit, symbol u, is equal to 1.6605×10^{-27} kg.

1 Show that the mass decreases in this reaction.

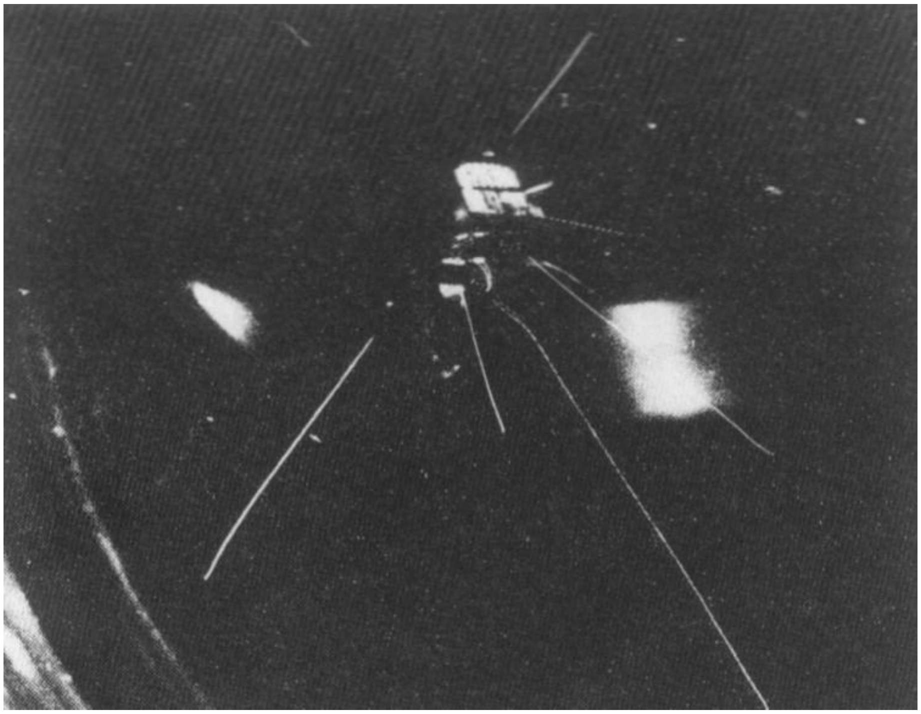
$$^{1}_{1}\text{H} + {}^{7}_{3}\text{Li} \rightarrow {}^{4}_{2}\,\text{He} + {}^{4}_{2}\text{He}.$$

Calculate Δm in atomic mass units and in kilograms.

Change in energy

2 The energy of the protons was 800 000 electron volts (800 keV). The lithium was in solid form so the nuclei would only have been vibrating due to thermal energy, less than an electron volt.

The reaction was captured in this photograph:



Two pairs of alpha particles, emerging in opposite directions, can be seen in the photograph.

From the range of the tracks through the cloud chamber the energy of the alpha particles was measured to be 8.5 MeV each.

Show that the total kinetic energy of the particles increases, and calculate ΔE in MeV and in joules.

3 If the increase in kinetic energy comes from the decrease in rest energy you should expect $\Delta E = \Delta mc^2$. Calculate the ratio of the change in kinetic energy to the change in mass $\Delta E/\Delta m$ in J kg^{-1}.

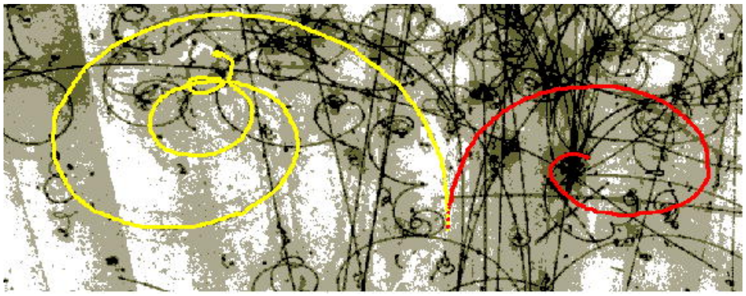
4 Show that the value of the ratio $\Delta E/\Delta m$ is approximately consistent with the relationship $\Delta E = \Delta mc^2$.

The large value of c^2 (9×10^{16} J kg^{-1}: use this value from now on in calculations) means that a small change in mass represents a vast change in rest energy. This relationship between mass and energy is why particle physicists measure masses in MeV / c^2; any unit of energy divided by c^2 is a unit of mass.

Creating massive particles

Energy is ‘materialised’ in matter–antimatter production. A photon of electromagnetic radiation can produce an electron and a positron. In this case, the energy of the photon vanishes and the rest energy of the particles appears. (This reaction needs to take place near to the nucleus of a heavy atom to conserve momentum but this is not going to affect your calculations here.)

In this bubble chamber photograph a photon enters from the bottom. It is uncharged and so produces no observable track. After some distance the photon disappears and produces the electron–positron pair. These two charged particles ionise the liquid in the chamber and bubbles form near the ions and are photographed.



In this case the chamber is filled with liquid hydrogen mixed with liquid neon. It is held under pressure which is released just as the particles enter the chamber to encourage bubbles to form and enlarge near the ions.

5 The bubble chamber is in a magnetic field, so charged particles bend due to the force Bqv on a moving charge. How does the photograph show that the two particles have opposite charges?

6 The mass of the electron is 5.5×10^{-4} u. What is the minimum energy photon that will produce an electron–positron pair? From what part of the electromagnetic spectrum is this? (Planck constant $h = 6.63 \times 10^{-34}$ J Hz^{-1}.)

Nuclear binding energy

If protons and neutrons (together known as nucleons) are bound together in a nucleus, the bound nucleus must have less energy than the nucleons of which it is made. That is, the rest energy of the nucleus must be less than the sum of the rest energies of its nucleons. In turn, this means that the mass of the nucleus must be less than the sum of the masses of its nucleons.

The simplest compound nucleus is the deuteron, the nucleus of hydrogen-2. It consists of a proton and a neutron bound together by the strong nuclear force. The masses of these particles are:

- proton: 1.0073 u
- neutron: 1.0087 u
- deuteron: 2.0136 u.

7 Calculate the difference in mass between a deuteron and one proton and one neutron.

8 Calculate the binding energy of the deuteron in J and in MeV.

9 Calculate the binding energy per nucleon of the deuteron.

10 Express the difference in mass as a percentage of the sum of the masses of the proton and neutron.

Mass change in nuclear fission

A possible reaction for the nuclear fission of uranium-235 is:

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{133}_{51}Sb + ^{99}_{41}Nb + 4^{1}_{0}n.$

The masses of the particles are

- U-235 = 235.0439 u
- Sb-133 = 132.9152 u
- Nb-99 = 98.9116 u
- neutron (n) = 1.0087 u.

11 Show that the energy change per atom of uranium is about 200 MeV and calculate $\Delta m/m$.

Summary

Einstein's famous equation $E_{rest} = mc^2$ reveals a Universe that is not as simple as it seems at first sight. The mass of a particle is generally a very large part of its total energy. The existence of rest energy was not suspected until after Einstein had predicted it, because the change in mass is usually so small, because changes in energy are usually a small fraction of the rest energy. Only in nuclear reactions where $\Delta m/m \sim 0.1\%$ or more are you able to see the change in mass, accompanied by what appears to be a huge change in energy.

Hints

1 Compare masses of H plus Li with mass of two He nuclei.

2 Two 8.5 MeV alpha particles come out, but one 800 keV proton goes in.

3 Compare the answers to questions 1 and 2.

4 Don't expect to get exactly the speed of light. Remember to take the square root of c^2!

5 What is the difference between forces F and - F?

6 Start with the mass of an electron in atomic mass units. Convert to kilograms. Write down the mass of an electron–positron pair. Use $E_{rest} = mc^2$ to get the rest energy of the pair in joules. Then use E = hf.

7 Do this one in the same way as question 1.

8 $E_{rest} = mc^2$ again. But now use the electron charge to get to electron volts and MeV.

9 How many nucleons in a deuteron?

10 Best to take the difference as a fraction of the mass before.

11 Add up before and after masses in atomic mass units first. Don't forget there's one extra neutron to start with and four extra neutrons afterwards. Then convert mass changes first to joules and then to MeV.

Practical advice

These questions practise the use of the relation between rest energy and mass in various contexts: nuclear transmutation, creation of particle–antiparticle pairs, nuclear binding and nuclear fission.

You may need to select only certain groups of questions, depending on what the class has covered. Alternatively, the whole set could be used for revision.

Some of the questions make extra demands in frequent changes of units, between atomic mass units, kilograms, joules and electron volts or MeV. You may well need to give extra help here.

Note the consistent use of the term 'rest energy'. The rest energy is treated as part of the total energy. It manifests itself in the mass of a particle. If mass is measured in kilograms and energy in joules, then the conversion is $E_{rest} = mc^2$. Remember that the mass, an invariant, is a physical property of a particle independent of frame of reference.

The questions bring out the fact that the rest energy is a very large fraction of the total energy, in many cases.

Alternative approaches

You may want to show that energies involved in everyday processes involve negligible changes in mass. The calculations of percentage change of mass in the questions here provide a starting point.

Social and human context

It is not possible to ignore the consequences for war and peace of the possibility of tapping these very large sources of energy.

Answers and worked solutions

1.

$$\text{Mass of H plus Li} = 1.0073\text{ u} + 7.0160\text{ u} = 8.0233\text{ u}$$

$$\text{Mass of two He} = 2 \times 4.0015\text{ u} = 8.0030\text{ u}$$

$$\text{Difference } \Delta m = 8.0030\text{ u} - 8.0233\text{ u} = -0.0203\text{ u}$$

So we can find the mass difference in kg:

$$\Delta m = -0.0203\text{ u} \times 1.6605 \times 10^{-27}\text{ kg} = -3.3708 \times 10^{-29}\text{ kg}$$

2.

Increase in energy:

$$\Delta E = 2 \times 8.5\text{ MeV} - 0.8\text{ MeV} = 16.2\text{ MeV}$$

In joules:

$$\Delta E = (16.2 \times 10^{6}\text{ eV}) \times (1.6 \times 10^{-19}\text{ J eV}^{-1}) = 2.6 \times 10^{-12}\text{ J}$$

3.

$$\frac{\Delta E}{\Delta m} = \frac{2.60 \times 10^{-12} \text{ J}}{3.37 \times 10^{-29} \text{ kg}} = 7.7 \times 10^{16} \text{ J kg}^{-1}.$$

4. If $\Delta E = \Delta mc^2$, then $c^2 = 7.7 \times 10^{16}$ J kg^{-1}, so c = 2.8×10^8 m s^{-1}.

5. The force on a moving charged particle is Bqv. If the charge q changes sign, the direction of the force is reversed, so the curvature is opposite.

6. The mass of an electron or positron is equal to:

$$(5.5 \times 10^{-4} \text{ u}) \times (1.66 \times 10^{-27} \text{ kg}) = 9.1 \times 10^{-31} \text{ kg}.$$

From $E_{rest} = mc^2$, the rest energy of an electron–positron pair is:

$$E_{\text{rest}} = 2 \times 9.1 \times 10^{-31} \text{ kg} \times (3 \times 10^8 \text{ m s}^{-1})^2 = 1.6 \times 10^{-13} \text{ J}.$$

If this energy is supplied by a photon of energy E = hf, then:

$$f = \frac{1.6 \times 10^{-13} \text{ J}}{6.63 \times 10^{-34} \text{ J Hz}^{-1}} = 2.5 \times 10^{20} \text{ Hz}.$$

This is the frequency of a gamma ray.

7. The mass difference is:

$$2.0136 \text{ u} - (1.0073 \text{ u} + 1.0087 \text{ u}) = -0.0024 \text{ u}.$$

I n kg the mass difference is:

$$-0.0024 \text{ u} \times (1.66 \times 10^{-27} \text{ kg}) = -3.98 \times 10^{-30} \text{ kg}.$$

8.

$$\begin{aligned} \text{Binding energy} &= -3.98 \times 10^{-30} \text{ kg} \times (3 \times 10^8 \text{ m s}^{-1})^2 \\ &= -3.58 \times 10^{-13} \text{ J} \\ &= -\frac{3.58 \times 10^{-13} \text{ J}}{1.6 \times 10^{-19} \text{ J eV}^{-1}} = -2.2 \times 10^6 \text{ eV} \\ &= -2.2 \text{ MeV}. \end{aligned}$$

9. The deuteron has two nucleons so the binding energy per nucleon is

–2.2 MeV / 2 = –1.1 MeV.

10. As a percentage the mass difference is equal to:

$$\frac{0.0024 \text{ u}}{1.0073 \text{ u} + 1.0087 \text{ u}} = 1.2 \times 10^{-3} \times 100 = 0.1\% \text{ (approximately)}$$

11.

$$\text{Mass after} = 132.9152 \text{ u} + 98.9116 \text{ u} + (4 \times 1.0087 \text{ u}) = 235.8616 \text{ u}$$

$\text{Mass difference} = 236.0526\ \text{u} - 235.8616\ \text{u} = 0.191\ \text{u}.$

$$\text{Change in rest energy} = \frac{0.191\ \text{u} \times (1.66 \times 10^{-27}\ \text{kg}) \times (3 \times 10^{8}\ \text{m s}^{-1})^{2}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = 1.78 \times 10^{8}\ \text{eV} = 178\ \text{MeV}.$$

The ratio is given by:

$\Delta m / m = 0.191\ \text{u} / 236\ \text{u} = 8.1 \times 10^{-4} \sim 0.1\%.$

External reference

This activity is taken from Advancing Physics chapter 18, 200S

TAP 525-2: Finding binding energy

Binding energy of carbon-12 nucleus

Mass of carbon-12 atom

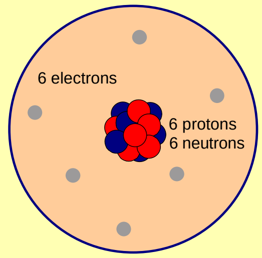
1 atomic mass unit u
= 1/12 of mass of C-12 atom
1 u = 1.66056×10^{-27} kg

mass of C-12 atom = 12.0 u

Mass of 6 electrons

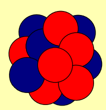
mass of electron
= 9.1095×10^{-31} kg
= 0.000549 u

mass of 6 electrons = 0.0033 u



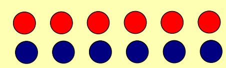
Calculate mass of carbon-12 nucleus

mass of carbon-12 nucleus
= mass of carbon-12 atom
– mass of 6 electrons



mass of carbon-12 nucleus
= (12.000 – 0.0033) u
= 11.9967 u

Calculate mass of all the protons and neutrons



mass of proton
= 1.67265×10^{-27} kg
= 1.00728 u

mass of neutron
= 1.67495×10^{-27} kg
= 1.00866 u

mass of 6 protons and 6 neutrons
= 6 (1.00728 + 1.00866) u
= 12.0956 u

Difference in mass

= mass of carbon-12 nucleus – mass of protons and neutrons
= (11.9967 – 12.0956) u
= – 0.0989 u

Binding energy

in mass units:
= – 0.0989 u
= -1.643×10^{-28} kg

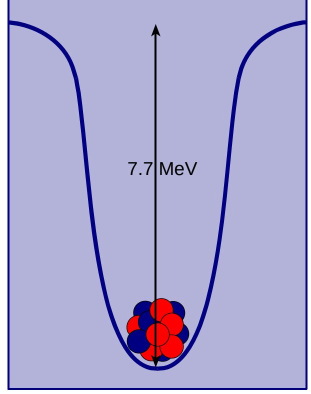
$E_{rest} = mc^2$

in energy units:
= -1.477×10^{-11} J
= – 92.16 MeV

Binding energy per nucleon

–92.16 MeV for 12 nucleons
= – 7.7 MeV per nucleon

Energy well for carbon-12 is 7.7 MeV per nucleon deep



Binding energy of a nucleus is the difference between its mass and the sum of the masses of its neutrons and protons

Practical advice

This diagram is reproduced here so that you can use it for discussion with your class.

External reference

This activity is taken from Advancing Physics chapter 18, 60P

TAP 525-3: Fusion in a kettle?

A change of scale

When you are confident with basic calculations of fission and fusion energy changes, you should work through these questions that try to put the energies of these changes into a more human scale for you. You will also need to understand the conversion of atomic mass units to energy and the meaning of the term 'electron volt'.

Try these

One of the reactions that fuel the stars is the fusion of two protons to give deuterium. In turn the deuterium goes through a series of reactions, the end product being helium. This is also a process that releases energy. In this question you are asked to consider the energy that would be released if all the deuterium in the water contained in an electric kettle were to be converted by fusion into helium.

The kettle contains 1 litre of water. The data you need are listed below.

1 atomic mass unit (u) = 931 MeV

1 eV = 1.6×10^{-19} J

$N_A = 6.02 \times 10^{23}$ mol^{-1}

Particle	Mass / u
${}^{1}_{1}H$	1.007 825
${}^{2}_{1}H$	2.014 102
${}^{3}_{2}He$	3.016 030
${}^{1}_{0}n$	1.008 665

1 Two deuterium nuclei ${}^{2}_{1}H$ can fuse to give one nucleus of helium ${}^{3}_{2}He$ with the ejection of one other particle. Write down the balanced equation that represents this reaction.

2 Calculate the mass change that occurs in this reaction.

3 Convert this energy into joules.

This gives you the energy released when two deuterium nuclei fuse. The next steps take you through the calculation of the total energy released if all the deuterium in the kettle water were to fuse to make helium-3. The ratio of deuterium atoms to hydrogen in water is roughly 1 to 7000.

4 What is the mass of 1 mole of water (H = 1 u; O = 16 u roughly)?

5 How many moles of water are contained in the litre?

6 How many molecules of water (H_2O) are in the kettle?

7 How many molecules of deuterium oxide (D_2O) are in the kettle?

8 Each heavy water molecule has two atoms of deuterium; what total energy is released if all the deuterium in the kettle is converted to helium-3?

Now to put this number in a new perspective. It requires 4200 J to increase the temperature of 1kg of water by 1K.

9 How many litres of water could be heated through 100 K by the fusion energy you calculated in question 8?

Hints

1 It is important to consider the atomic electrons in this equation. You begin with two, one for each hydrogen. How many electrons does an un-ionised atom of deuterium have? So what must one of the emitted particles be? This should lead you to the other particle.

2 The conversions you need are near the data table in the question.

4 The formula of water shows that there are two hydrogen atoms and one oxygen for each water molecule.

5 1 litre of water has a mass of 1 kg.

6 1 mole contains 6×10^{23} molecules of water.

Practical advice

These questions can be modified in many ways, not least by changing the homely example of a kettle to perhaps a bath full of water or even to Lake Windermere or the local reservoir.

Social and human context

The 6000 litres of heated water may not seem so significant until you realise that this has come from the fusion of deuterium which had an original volume of 0.15 cm^3.

Answers and worked solutions

1.

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$

2. Δm = (3.016 030 u + 1.008 665 u) – 2 × 2.014 102 u = – 0.0035 u

3. 0.003509 u × 931 × 10^6 eV u^{-1} × 1.6 × 10^{-19} J eV^{-1} = 5.23 × 10^{-13} J

4. 18 g

5. 1 litre of water has a mass of 1 kg.

   number of moles = 1000 g / 18 $gmol^{-1}$ = 56 mol

6. 56 mol × 6.02 × 10^{23} mol^{-1} = 3 × 10^{25}

7. (3.4 ×10^{25})/7000= 4.9 × 10^{21}

8. energy released = 4.9 ×10^{21} × (5.23 ×10^{-13} J) = 2.49 × 10^9 J

9. (2.49 ×10^9 J)/ (4200 J $kg^{-1}K^{-1}$ × 100K) = 6000 kg = 6000 litres

External reference

This activity is taken from Advancing Physics chapter 18, 260S

TAP 525- 4: A binding energy calculator

This model removes the drudgery from the calculation of binding energies. There are three sheets. Sheet 1 does the calculations.

4

4 The calculator

	numbers	masses / amu	energy / J
electrons	5	0.0025	
protons	5	5.0365	
neutrons	7	7.0609	
constitutents total		12.0999	
quoted mass of isotope		11.00931	
difference in mass		-1.09059	
energy released			-1.63E-10

Sheet 2 holds the common data – masses for neutrons, protons, electrons, the value of one atomic mass unit in kilograms and the speed of light

Masses of particles

name	symbol	mass / unified atomic
electron	${}^{0}_{-1}e$	0.0005
proton	${}^{1}_{1}p$	1.0073
neutron	${}^{1}_{0}n$	1.0087

Constants			
Speed of light	c	3.00E+08	m/s
1 amu		1.67E-27	kg

Sheet 3 holds a sample of isotope data.

Element	Z	A	Mass / amu
H	1	1	1.00783
He	2	4	4.00260
Li	3	7	7.01600
Be	4	9	9.01218
Be	5	11	11.00931
C	6	12	12.00000
N	7	14	14.00307
O	8	16	15.99491
F	9	19	18.99840
Ne	10	20	19.99244
Na	11	23	22.98980
Mg	12	24	23.98504
Al	13	27	26.98153
Si	14	28	27.97693
P	15	31	30.97376
Si	16	32	31.97207
Cl	17	35	34.96885
Ar	18	38	37.96272
K	19	39	38.96371
Ca	20	40	39.96259
Sc	21	45	44.95592
Ti	22	47	46.95180
V	23	51	50.94400
Cr	24	52	51.94050
Mn	25	55	54.93810
Fe	26	56	55.93490
Ni	28	58	57.93530
Co	27	59	58.93320
Cu	29	63	62.92980
Zn	30	64	63.92910
Ga	31	69	68.92570
Ge	32	74	73.92190
As	33	75	74.92160
Br	35	79	78.91830
Se	34	80	79.91650
Kr	36	82	81.91350
Rb	37	85	84.91170
Sr	38	88	87.90560
Y	39	89	88.90540
Zr	40	90	89.90430
Nb	41	93	92.90600
Mo	42	98	97.90550
Ru	44	102	101.90370
Rh	45	103	102.90480
Pd	46	106	105.90320
Ag	47	107	106.90509
Cd	48	114	113.90360
In	49	115	114.90410
Sn	50	118	117.90180
Sb	51	121	120.90380
In	53	127	126.90040
Te	52	130	129.90670
Xe	54	132	131.90420
Cs	55	133	132.90510
Ba	56	138	137.90500
La	57	139	138.90610

What to do:

Enter the values for the chosen isotope into the pale yellow boxes in sheet 1, following the tips in the comment boxes, and the binding energy is calculated and displayed in the pale blue box.

You will need to double click on the boxes and have a computer running Excel

Practical advice

This is provided as a constructed calculator, together with some useful data.

External reference

This activity is taken from Advancing Physics chapter 18, File 30T

TAP 525-5: Binding energy of nuclei

Looking for patterns

You will use the data in a spreadsheet to calculate the binding energy of a set of nuclei. You will then produce a plot to show how the binding energy per nucleon varies with mass of the nucleus.

You will need

- ✓ computer running a spreadsheet
- ✓ data provided in spreadsheet format

Building the spreadsheet

Take a look at the four columns in the spreadsheet data. The first is simply the name of some of the stable elements. This is followed by a column showing the atomic number (*Z*, the number of protons in the nucleus) and a column giving the mass number (A, the total number of nucleons, i.e. protons plus neutrons). Finally there is a column giving the actual atomic mass. The units of this column are atomic mass units, which are defined as exactly one-twelfth of the mass of a carbon-12 atom. The atomic mass unit (u) is also called the unified atomic mass constant, and has a value of $1.660\ 5402 \times 10^{-27}$ kg.

Use this information to calculate the binding energy of each nucleus. The binding energy is simply the difference in energy between a nucleus and its constituent parts. This energy change can be measured as a change in the mass of the nucleus. A useful shortcut is that a mass difference of 1 atomic mass unit is equivalent to 931 MeV (million electron volts) of energy.

To find the binding energy you will need to subtract the mass of the constituents from the atomic mass. The constituents are Z protons, (A – Z) neutrons and Z electrons (electrons are included in the atomic mass). The masses of these in atomic mass units are:

- mass of neutron = 1.008 665 u
- mass of proton = 1.007 277 u
- mass of electron = 0.000 548 u

Create new columns in the spreadsheet giving the number of neutrons and the mass of the constituents. Now calculate the binding energy of the entire nucleus and the binding energy per nucleon. Plot this last quantity against mass number (not atomic number).

Double click on the chart below, you will need a computer running Excel.

Element	Z	A	Mass / u
H	1	1	1.00783
He	2	4	4.00260
Li	3	7	7.01600
Be	4	9	9.01218
Be	5	11	11.00931
C	6	12	12.00000
N	7	14	14.00307
O	8	16	15.99491
F	9	19	18.99840
Ne	10	20	19.99244
Na	11	23	22.98980
Mg	12	24	23.98504
Al	13	27	26.98153
Si	14	28	27.97693
P	15	31	30.97376
Si	16	32	31.97207
Cl	17	35	34.96885
Ar	18	38	37.96272
K	19	39	38.96371
Ca	20	40	39.96259
Sc	21	45	44.95592
Ti	22	47	46.95180
V	23	51	50.94400
Cr	24	52	51.94050
Mn	25	55	54.93810
Fe	26	56	55.93490
Ni	28	58	57.93530
Co	27	59	58.93320
Cu	29	63	62.92980
Zn	30	64	63.92910
Ga	31	69	68.92570
Ge	32	74	73.92190
As	33	75	74.92160
Br	35	79	78.91830
Se	34	80	79.91650
Kr	36	82	81.91350
Rb	37	85	84.91170
Sr	38	88	87.90560
Y	39	89	88.90540
Zr	40	90	89.90430
Nb	41	93	92.90600
Mo	42	98	97.90550
Ru	44	102	101.90370
Rh	45	103	102.90480
Pd	46	106	105.90320
Ag	47	107	106.90509
Cd	48	114	113.90360
In	49	115	114.90410
Sn	50	118	117.90180
Sb	51	121	120.90380
In	53	127	126.90040
Te	52	130	129.90670
Xe	54	132	131.90420
Cs	55	133	132.90510
Ba	56	138	137.90500
La	57	139	138.90610

particle	mass /u
neutron	1.008 665
proton	1.007 277
electron	0.000 548

There are four columns in the spreadsheet data.

- The name of some of the stable elements.
- The atomic number (Z, the number of protons in the nucleus).
- The mass number (A, the total number of nucleons: protons plus neutrons).
- The actual atomic mass. The units of this column are atomic mass units, which are defined as exactly one-twelfth of the mass of a carbon-12 atom. The atomic mass unit is also called the unified atomic mass constant, and has a value of $1.660\ 5402 \times 10^{-27}$ kg.

You will have

1. A spreadsheet giving the binding energy of a selection of nuclei.
2. A graph of binding energy per nucleon against mass number.

Practical advice

Only a selection of stable nuclei have been included, and the data have been pre-sorted so they are in mass number order rather than atomic number order, and should therefore produce a graph very readily. Students need to be encouraged to change the default settings in their spreadsheet to make the graph clearer and more easily read - an example from Excel is included here. There are some obvious spikes in the graph, which students should be encouraged to think about.

This chart is a springboard for discussing why binding energies are negative, why fission and fusion release energy and why certain nuclei are more stable than others. The chart given here indicates some of the key features.

Alternative approaches

Use the chart given and ask students to investigate different parts of it - the long slow slope showing where fission releases energy, the steeper slope where fusion releases energy and the spikes at ^{4}He, ^{12}C and ^{16}O. These are particularly important for stellar fusion.

Social and human context

It has often been claimed that our Universe is a fluke because the values of certain fundamental constants are closely tuned to values that produce a Universe we can live in. One of these claims is that the fusion of helium in stars to produce carbon and hence all the heavier elements of which we are made requires a lucky coincidence of energy levels between ^{4}He, ^{8}Be (which is unstable and forms for a short time) and ^{12}C. However, a glance at the chart shows that elements such as ^{12}C and ^{16}O are very close to being clusters of helium nuclei so it is, perhaps, no surprise that the relevant energy levels are close to coincidence. A good reference on this, and other aspects of basic laws, is:

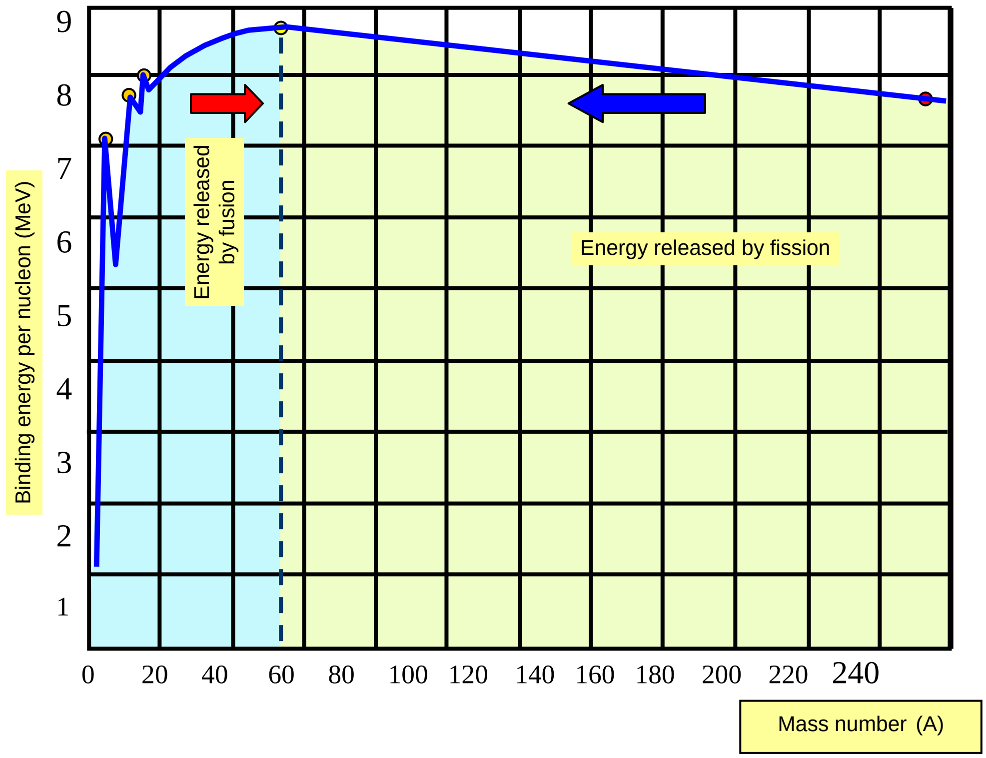
Dreams of a Final Theory by Steven Weinberg (published by Vintage).

External reference

This activity is taken from Advancing Physics chapter 18, 140s

TAP 525- 6: Binding energy per nucleon

The graph below shows the binding energy per nucleon against nucleon number. Elements with a high binding energy per nucleon are very difficult to break up. Iron 56 has the highest binding energy per nucleon of any element and this which explains why there is so much of it in the universe.



The part of the curve to the left shows that two light elements can produce energy by fusion while the part of the curve to the right shows that a heavy element can produce energy by fission.

Therefore if a reaction takes place where the products are closer to the base then the original nucleus (nuclei) then energy is given out.

For helium the binding energy per nucleon is 28.3/4 = 7.1 MeV.

The helium nucleus has a high binding energy per nucleon and is more stable than some of the other nuclei close to it in the periodic table.

Some of the binding energies per nucleon for some common elements are shown in the following table.

Element	Mass of nucleons (u)	Nuclear Mass (u)	Binding Energy (MeV)	Binding Energy per Nucleon (MeV)
Deuterium	2.01594	2.01355	2.23	1.12
Helium 4	4.03188	4.00151	28.29	7.07
Lithium 7	7.05649	7.01336	40.15	5.74
Beryllium 9	9.07243	9.00999	58.13	6.46
Iron 56	56.44913	55.92069	492.24	8.79
Silver 107	107.86187	106.87934	915.23	8.55
Iodine 127	128.02684	126.87544	1072.53	8.45
Lead 206	207.67109	205.92952	1622.27	7.88
Polonium 210	211.70297	209.93683	1645.16	7.83
Uranium 235	236.90849	234.99351	1783.80	7.59
Uranium 238	239.93448	238.00037	1801.63	7.57

A very useful web site containing a huge nuclear database is to be found at:

http://nucleardata.nuclear.lu.se

It may be more helpful to consider the binding energy per nucleon diagram in the form shown in Figure 2 where reactions tend to move the nuclei towards the valley at the bottom of the curve. (In this case note that the binding energies per nucleon are given as negative values).

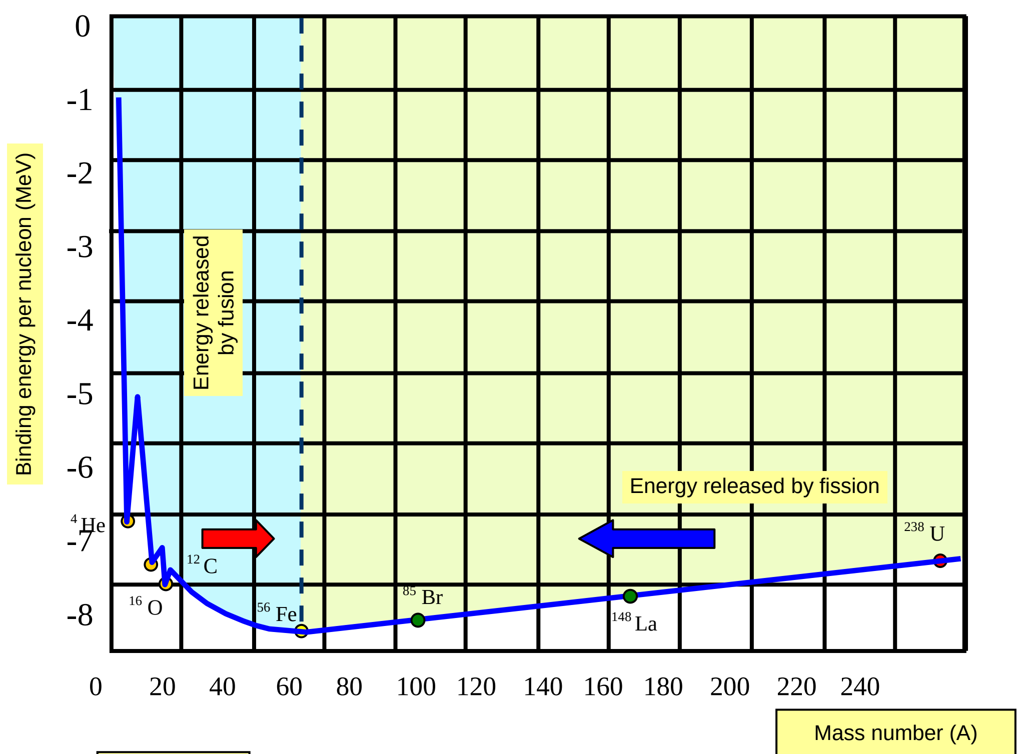


Figure 2

External reference

This activity is taken from Resourceful Physics