TAP 522- 4: Rutherford scattering: Energy and closest approach

Scattering of alpha particles

Rutherford did not have a particle accelerator. Instead he used alpha particles, typically of energy 5 MeV, from radioactive decay. These questions are about how close an alpha particle can get to different nuclei.

An alpha particle has charge 2e, where $e = 1.60 \times 10^{-19}$ C. A nucleus has charge Ze, where Z is the number of protons in the nucleus (and the number of electrons in the atom). The electrical potential energy of the two charges at a distance r is:

$$\text{electrical potential energy} = \frac{2e \times Ze}{4\pi\varepsilon_0 r}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 J^{-1} m^{-1}.

The electrical potential energy in electron volts is obtained by dividing by 1.60×10^{-19} J eV^{-1}

Calculating the potential energy

1.   Show that the units of energy from the expression

$$\text{electrical potential energy} = \frac{2e \times Ze}{4\pi\varepsilon_0 r}$$
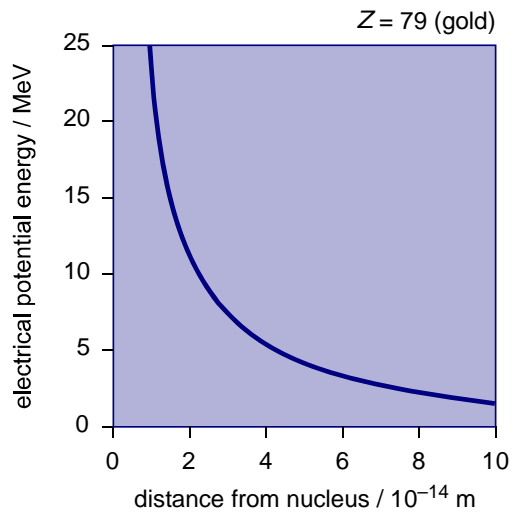
are joules.

2.   Show that the energy in MeV is given by

$$\frac{2Ze}{4\pi\varepsilon_0 r} \times 10^{-6}.$$

Alpha scattering by gold

This graph shows the energy in MeV of an alpha particle at distances r from a gold nucleus, Z = 79.

Approach of alpha particle to nucleus

Z = 79 (gold)

electrical potential energy / MeV

distance from nucleus / 10^{-14} m

3.   Make an arithmetical check to show that at distance r = 1.0×10^{-14} m the electrical potential energy is between 20 MeV and 25 MeV, as shown by the graph.

4.   How does the electrical potential energy change if the distance r is doubled?

5.   From the graph, at what distance *r* will an alpha particle with initial kinetic energy 5 MeV colliding head-on with the nucleus, come to rest momentarily?

6.   From the graph, at what distance r will a 5 MeV alpha particle have lost half its initial kinetic energy?

7.   From the graph, what energy would an alpha particle need to approach as close as 2.0×10^{-14} m in a head-on collision?

Alpha scattering by tin

The next graph shows, on the same scale as before, the potential energy of an alpha particle near a nucleus of tin, Z = 50.

Approach of alpha particle to nucleus

Z = 50 (tin)
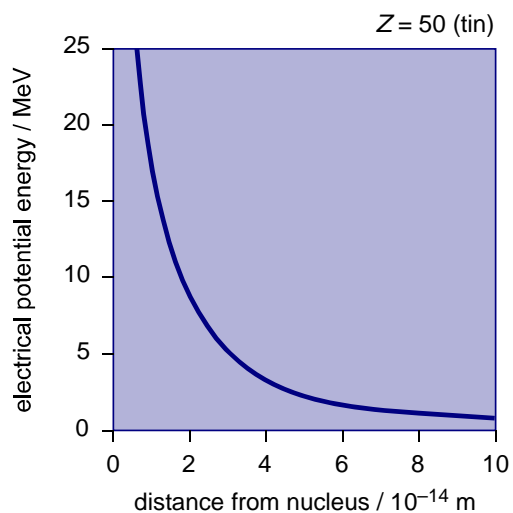
8.   Why are the values of the potential energy smaller at the same values of r in this second graph?

9.   At r = 5.0×10^{-14} m the electrical potential energies of an alpha particle are 4.55 MeV for gold, Z = 79 and 2.88 MeV for tin, Z = 50. Explain the ratio, 1.58, of the two energies.
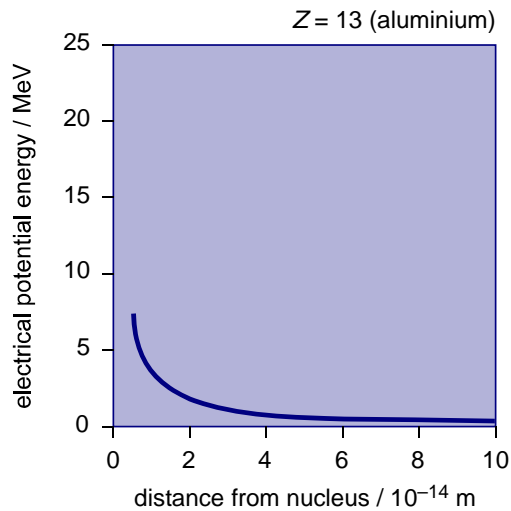
10.   Approximately how close can a 5 MeV alpha particle get to a tin nucleus, in a head-on collision?

Alpha scattering by aluminium

The next graph shows the potential energy of an alpha particle close to an aluminium nucleus,

Z = 13.

Approach of alpha particle to nucleus

Z = 13 (aluminium)

electrical potential energy / MeV

distance from nucleus / 10^{-14} m

11.   From the graph, how close could a 5 MeV alpha particle get to a nucleus of charge?

Z = 13, in a head-on collision?

12.   The radius of an aluminium nucleus is approximately 3×10^{-15} m. Does a 5 MeV alpha particle get close to the nucleus, compared with the dimensions of the nucleus itself? Could the pattern of scattering be affected?

Heavy ion colliders

Recently, to investigate very high-energy collisions, accelerators have been used to make head-on collisions between lead nuclei travelling in opposite directions.

13.   How much kinetic energy is needed to get two lead nuclei, *Z* = 82,

within 1.0×10^{-14} m of one another? (Assume that electrical forces are the only forces operating.)

Hints

1.   Treat units like algebraic quantities in the expression for potential energy.

2.   Remember that the charge e coulomb is also the conversion joule per electron volt.

3. Substitute values in the equation for potential energy.

4. Remember 1/r.

5. Read approximately from the graph.

6. Read approximately from the graph.

7. Read approximately from the graph.

8. Look at the equation for electrical potential energy.

9. Try the ratio 79/50.

10. Read approximately from the graph.

11. Read approximately from the graph.

12. Remember that 10^{-15} is 1/10 of 10^{-14}.

13. Substitute in the expression for electrical potential energy. Or start with the answer to question 3.

Practical advice

The questions focus on the distance of closest approach of an alpha particle to a nucleus. The approach is through the shape of the 1/r curve of electric potential energy, and the way the curve varies with radius and charge.

Alternative approaches

Students could explore the electric potential energy close to a nucleus, using a spreadsheet.

Social and human context

With hindsight, Rutherford was very clever to have managed without a particle accelerator. But how could he have raised the money to build one without knowing what he would find?

Answers and worked solutions

1. The units are:

$$\frac{C \times C}{C^2\, J^{-1}\, m^{-1} \times m} = J.$$

2. In the expression

$$\text{electrical potential energy} = \frac{2e \times Ze}{4\pi\varepsilon_0 r}$$

dividing by e gives

$$\frac{2Ze}{4\pi\varepsilon_0 r}$$

for the energy in eV. Multiply by 10^{-6} to get the energy in MeV.

3. Substituting values gives

$$E_P = \frac{2 \times 79 \times 1.6 \times 10^{-19}\, C}{4\pi \times 8.85 \times 10^{-12}\, C^2\, J^{-1}\, m^{-1} \times 1.0 \times 10^{-14}\, m} \times 10^{-6} = 22.7\, \text{MeV}.$$

4. Halves, because the potential energy is proportional to 1/r.

5. About 4.6×10^{-14} m, where the graph reaches 5 MeV.

6. Just less than 10.0×10^{-14} m.

7. About 12 MeV.

8. The charge on the nucleus is smaller, so the potential energy is smaller in the same ratio.

9. The ratio of the charges, 79 / 50 = 1.58.

10. About 3×10^{-14} m.

11. About 0.75×10^{-14} m.

12. The alpha particle approaches to about 2.5 times the radius of the nucleus. Attractive forces between nucleons might begin to be important, and modify the scattering.

13. Inserting values:

$$E_{P} = \frac{82 \times 82 \times 1.6 \times 10^{-19} \text{ C}}{4\pi \times 8.85 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1} \times 1.0 \times 10^{-14} \text{ m}} \times 10^{-6} = 967 \text{ MeV.}$$

External reference

This activity is taken from Advancing Physics chapter 17, 70S