

## Episode 522: The size of the nucleus

Having established the existence of the nucleus, you can now consider experimental evidence for its size, starting from the Rutherford experiment.

### Summary

**Discussion + worked example: Size of nucleus (15 minutes)**

**Discussion: Atomic and nucleus size (10 minutes)**

**Student questions: Forces and closest approach (30 minutes)**

**Discussion: Atomic number and the charge on a nucleus (5 minutes)**

**Discussion: Upper limit of nuclear size (30 Minutes)**

**Discussion: A puzzle for a future lesson (5 minutes)**

### Discussion + worked example:

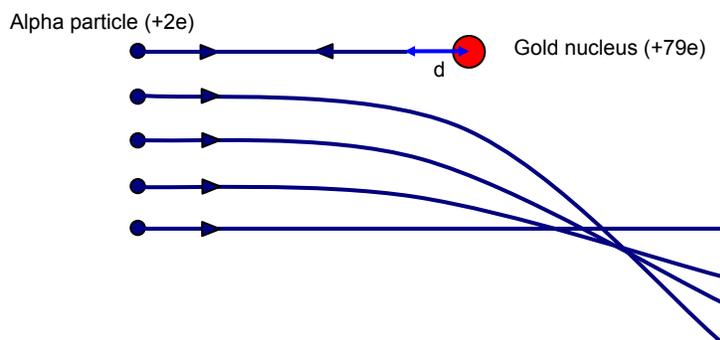
#### Size of the nucleus

You can get an idea of the possible size of the nucleus by thinking about Rutherford's experiment. Ask: What impact parameter will result in the  $\alpha$  particle getting closest to the nucleus? (A 'head-on' collision with  $p = 0$ .)

The principle of the conservation of energy is used to calculate the distance of closest approach as a measure for the size of a nucleus. Understanding the calculation that follows depends upon whether the students have covered electric potential and fields. Alternatively it serves as good revision.

TAP 522-1: Alpha particle scattering – distance of closest approach

TAP 522-2: Distance of closest approach.



When the  $\alpha$  is brought momentarily to rest ("having climbed as far as it can up the electrostatic hill") the work done in bringing it to rest will just equal its initial kinetic energy. When the speed and hence the kinetic energy is zero, all the energy is now electrostatic potential energy.

If the  $\alpha$  momentarily stops when at a distance  $d$  from the (centre of) the nucleus of charge  $Ze$ , its electrical potential energy is

$$E_{\alpha} = \frac{1}{4\pi\epsilon_0} 2e \frac{Ze}{d}$$

This equals the initial kinetic energy of the  $\alpha$  particle. Rutherford used an  $\alpha$  source given to him by Madame Curie. The  $\alpha$  energy was  $\sim 7.7\text{MeV}$ .

For gold,  $Z = 79$ . Solving gives  $d \sim 3 \times 10^{-14}\text{ m}$ . Compare this with the diameter of gold atoms  $\sim 3 \times 10^{-10}\text{ m}$ . So a nucleus is at least 10 000 times smaller than an atom. It is important to emphasise that this calculation gives an **upper limit** on the size of the gold nucleus; we cannot say that the alpha particle *touches* the nucleus; a more energetic  $\alpha$  might get closer still.

An atom is mostly 'empty' (which is why most  $\alpha$ s went straight through – any electrons would hardly impede the relatively massive' high speed  $\alpha$ ).

## Discussion:

### Atomic and nucleus size

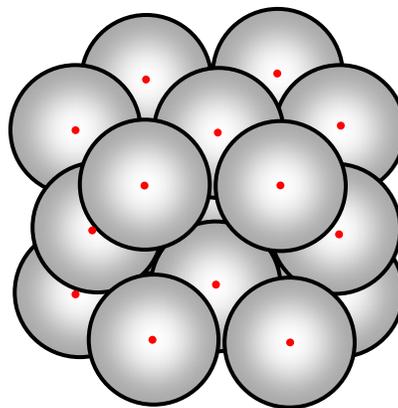
Ask your students to suggest a scale model of the nuclear atom. For example: if a nucleus was 1 mm diameter, an atom would be 10,000 times larger or 10 m in diameter. Choose a suitable position for a 1 mm nucleus (a small ball bearing or ball of Blu-tac). Pace out 5 m (five large steps) to the edge of the atom where the electrons are. NB: textbook diagrams of an atom with a nucleus are not drawn to scale.

Reinforce an accurate picture by getting a student to stand up as a 'nucleus', estimate their 'girth' (40 cm?) and ask where another student would have stand to be at the edge of the 'atom'. ( $10^4$  times 40 cm = 4000 m, so the radius of this "atom" is 2 km! Check with a local map to find a named location that students will recognize that is 2 km away.

Further reinforcement: in a solid where atoms are close packed, the distance between adjacent nuclei  $\sim$  the size of an atom, i.e. equivalent to two students standing 4 km apart!

So it's quite amazing that any  $\alpha$ s would 'hit' a nucleus at all! Both are a similar size. Cross sectional area presented by a nucleus  $\sim \text{radius}^2 = \sim 10^{-28}\text{ m}^2$ .

Ask: How would you expect the number of reflected  $\alpha$ s to depend upon the thickness of the metal foil containing the target nuclei? (Imagine the gold atoms in layers, chance of a deflection increases with thickness, but absorption on the way in or back out of the increasingly thick foil will eventually prevent any further increase in the number reflected *and detected*).



(Diagram: resourcefulphysics.org)

It is of great help if your students can recall the following orders of magnitude:

Radius of atomic nucleus  $\sim 10^{-14}\text{ m}$

Radius of atom  $\sim 10^{-10}\text{ m}$

## Student questions:

### Forces and closest approach

TAP 522-3: Rutherford scattering: Directions of forces

TAP 522-4: Rutherford scattering: Energy and closest approach

## Discussion

### Atomic number and the charge on a nucleus

Rutherford used his data to find the charge of the gold target nucleus. Further experiments to find the charge of Cu, Ag and Pt foils gave:

	atomic number	$\alpha$ scattering experiment
Cu	29	$29.3 \times e$
Ag	47	$46.3 \times e$
Pt	78	$77.4 \times e$

So the electric charge on a nucleus is given by the atomic number  $\times e$ , i.e.  $Ze$ . With one exception (hydrogen, H-1),  $Z$  is always less than the atomic mass number. So what accounts for the difference? The atom must be electrically neutral. Rutherford proposed the neutron.

## Discussion

### Upper limit of nuclear size

Recall that Rutherford's analysis gives an upper limit on the size of the nucleus ( $d \sim 1/(\alpha$  particle energy)). The size you measure depends upon the energy of the  $\alpha$  particle you use. So we need another approach to find the size of a gold nucleus. Can you think of a better particle to probe the size of a nucleus? (The neutron – being uncharged it will get closer.)

Another technique is the deep inelastic scattering of electrons. Refer back if you have already covered the wave nature of particles ('de Broglie waves'  $\lambda = h/p$ ), or this topic can be inserted here if desired. The electron diffraction apparatus has a basic similarity with  $\alpha$  particle scattering. The electrons are fired at a thin film – in this case of graphite.

Rutherford was fortunate that the de Broglie wavelength of the  $\alpha$  particles (unknown to him) was quite small, and the coulomb repulsion stops  $\alpha$ s getting too close – otherwise diffraction effects would have 'confused' the data! (Try the calculation if you have already covered  $\lambda = h/p$ .)

TAP 506-2: Interpreting electron diffraction patterns

TAP 522-5: Deep inelastic scattering

TAP 522-6: Electrons measure the size of nuclei

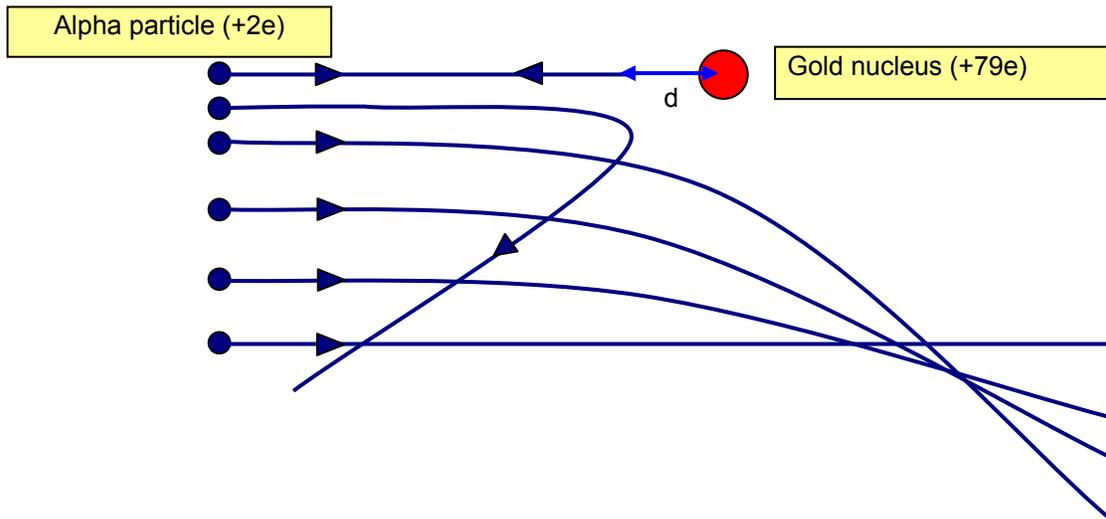
**Discussion:**

**A puzzle for a future lesson**

There is a fundamental problem with Rutherford's model. Ask your class: How can an atom with a central nucleus can be stable – why doesn't it collapse? According to classical electrodynamics, the electrons should radiate energy as they orbit, and spiral inwards.

(It's good to leave a class with a puzzle for a future lesson.)

## TAP 522-1: Alpha particle scattering – distance of closest approach



If an alpha particle with a kinetic energy  $E_\alpha$  is fired directly towards a gold nucleus it will feel a repulsion that increases as it gets closer - climbing the potential hill surrounding the nucleus. When all the kinetic energy has been converted to potential energy the alpha particle (charge  $+2e$ ) has reached its distance of closest approach ( $d$ ) and comes to rest at that point

**Practical advice**

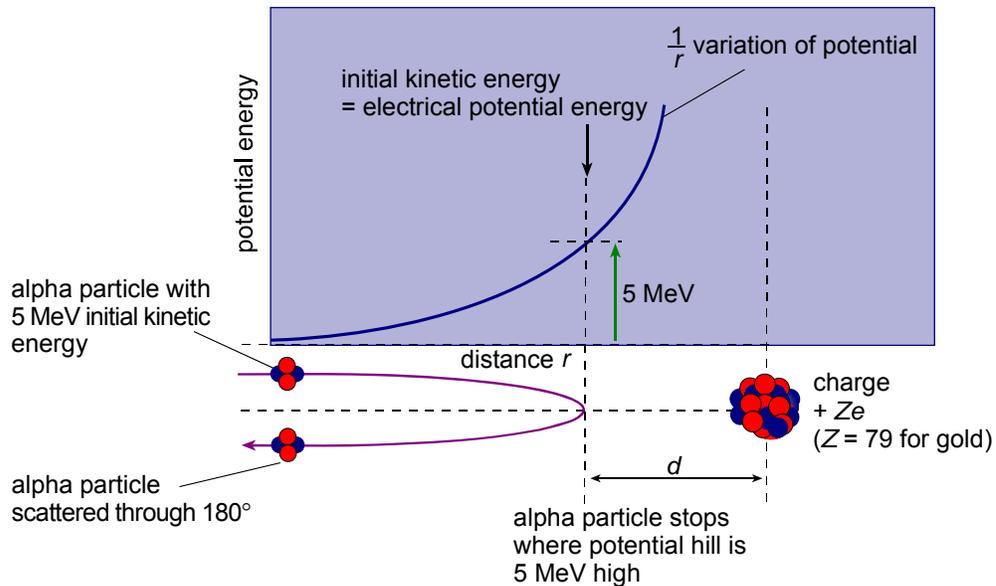
The diagram could be used as an OHT and discussed in class

**External Reference**

This activity is taken from Resourceful Physics

## TAP 522-2: Distance of closest approach

### Distance of closest approach



### Where does the alpha particle stop?

#### Initial kinetic energy

$$\begin{aligned} &= 5 \text{ MeV} \\ &= 5 \times 10^6 \text{ eV} \times 1.6 \times 10^{-19} \text{ J eV}^{-1} \\ &= 8.0 \times 10^{-13} \text{ J} \end{aligned}$$

#### Electrical potential energy

$$\begin{aligned} V &= \frac{+2Ze^2}{4\pi\epsilon_0 d} \\ Z &= 79, e = 1.6 \times 10^{-19} \text{ C}, \\ \epsilon_0 &= 8.9 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} \end{aligned}$$

Alpha particle stops where  
initial kinetic energy = electrical potential energy

$$8.0 \times 10^{-13} \text{ J} = \frac{+2Ze^2}{4\pi\epsilon_0 d}$$

substitute values of  $Z$ ,  $e$ ,  $\epsilon_0$ :

$$d = 4.5 \times 10^{-14} \text{ m}$$

Radius of gold nucleus must be less than of the order of  $10^{-14} \text{ m}$

Atoms are 10000 times larger than their nuclei

**Practical advice**

The diagram could be used as an OHT and discussed in class

**External reference**

This activity is taken from Advancing Physics chapter 17, 1300

## TAP 522- 3: Rutherford scattering: directions of forces

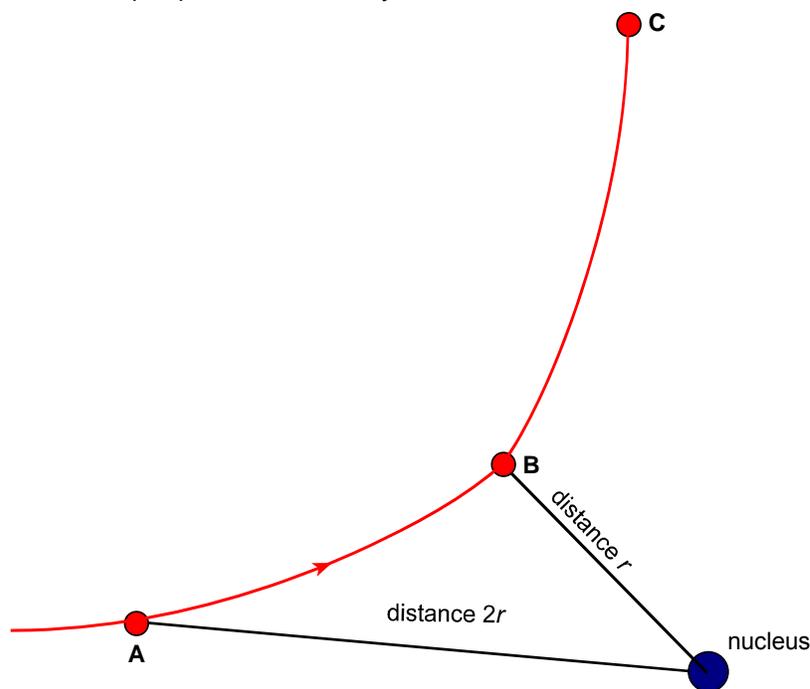
### Scattering of alpha particles

Rutherford did not have a particle accelerator. Instead he used alpha particles, typically of energy 5 MeV, from radioactive decay. These questions are about the force of the nucleus on the alpha particle.

An alpha particle has charge  $+2e$ , where  $e$  is the elementary unit of charge. A nucleus has charge  $+Ze$ , where  $Z$  is the number of protons in the nucleus (and the number of electrons in the atom).

### Directions of forces

Path of alpha particle scattered by nucleus



The diagram shows an alpha particle approaching a massive nucleus from A. Assume that the nucleus recoils negligibly as the alpha particle is scattered.

1. Add to the diagram an arrow showing the direction of the force on the alpha particle when it is at point A, approaching the nucleus. Label the arrow  $F_A$ .
2. Add to the diagram an arrow showing the direction of the force on the alpha particle when it is at point B, at its closest to the nucleus. Label the arrow  $F_B$ .
3. What is the ratio of the magnitudes of the two forces,  $F_A / F_B$  given the distances shown in the diagram?

4. Add to the diagram an arrow showing the direction of the force on the nucleus when the alpha particle is at point B. Label the arrow  $F_N$ . How does this force compare with the force  $F_A$  on the alpha particle at the same instant?
5. At which point, A, B or C, is the alpha particle travelling slowest?
6. At which point, A, B or C, is the alpha particle travelling fastest?
7. The nucleus does in fact recoil a little. Add an arrow labelled 'recoil' to show the direction of recoil you expect as a result of the passage of an alpha particle along the whole path shown.

### **Uphill–downhill**

The electrical potential gradient around the nucleus can be thought of as like the slope of a hill. Imagine that you are riding on the alpha particle as it goes by the nucleus. Are you riding uphill, downhill or momentarily along a contour of the hill:

8. At A?
9. At B?
10. At C?
11. Is the electric potential at B larger or smaller than the electric potential at A? By what factor?

## Practical advice

These are intended as simple 'start-up' questions for the discussion of alpha scattering. They are mainly qualitative, asking students to think about the direction of forces, and the nature of changes of kinetic and potential energy. The 'hill' metaphor for potential is exploited.

## Alternative approaches

Alpha particle orbits generated by a computer simulation could be studied is:

A Useful web site that includes a simulation of alpha particle scattering

[http://www-outreach.phy.cam.ac.uk/camphy/nucleus/nucleus\\_index.htm](http://www-outreach.phy.cam.ac.uk/camphy/nucleus/nucleus_index.htm)

The following sections are relevant to this topic

Plum pudding atoms,  $\alpha$  scattering, Geiger & Marsden, Shells off tissue paper, Nucleus

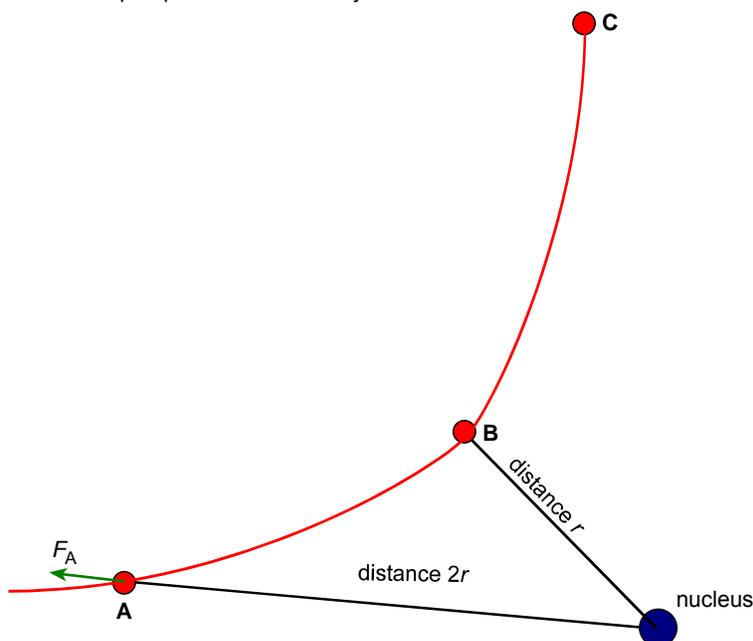
## Social and human context

The mathematical tools developed by the French mathematicians in the 1700s to deal with planetary orbits in the solar system were just as useful for alpha particle orbits under a repulsive force.

## Answers and worked solutions

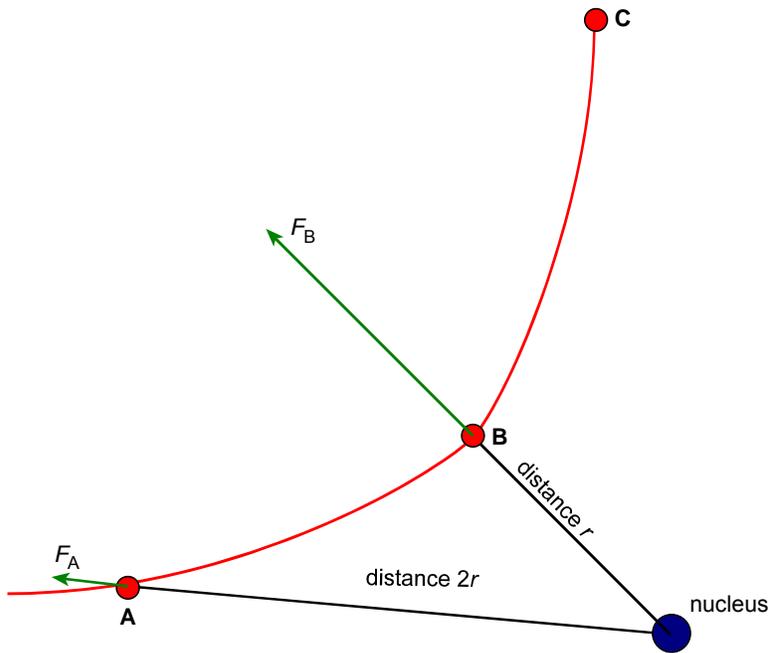
1. The repulsive force is along the line joining the alpha particle and the nucleus.

Path of alpha particle scattered by nucleus



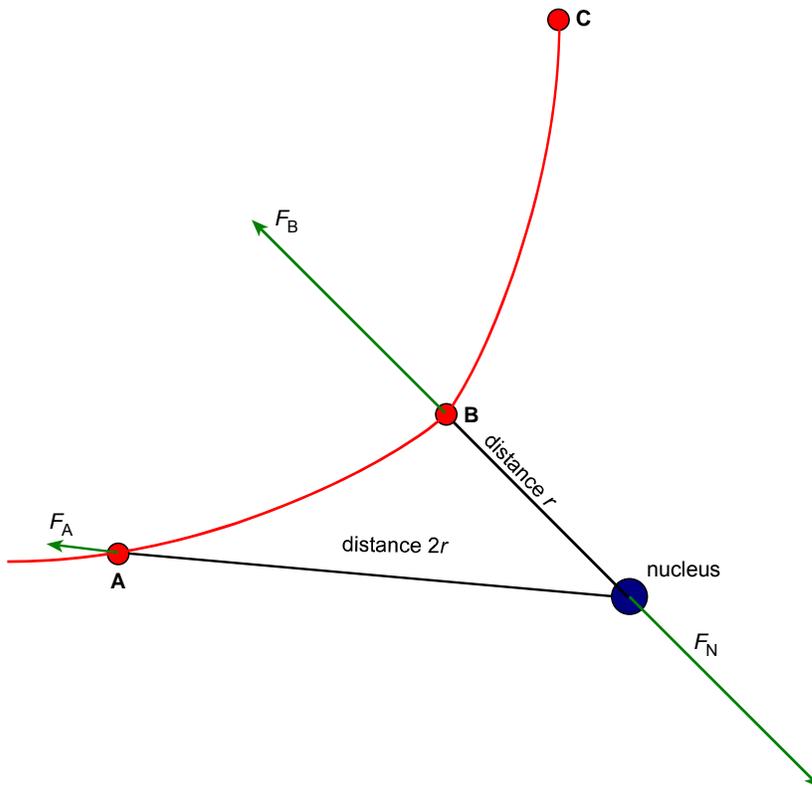
- The repulsive force is along the line joining the alpha particle and the nucleus.

Path of alpha particle scattered by nucleus



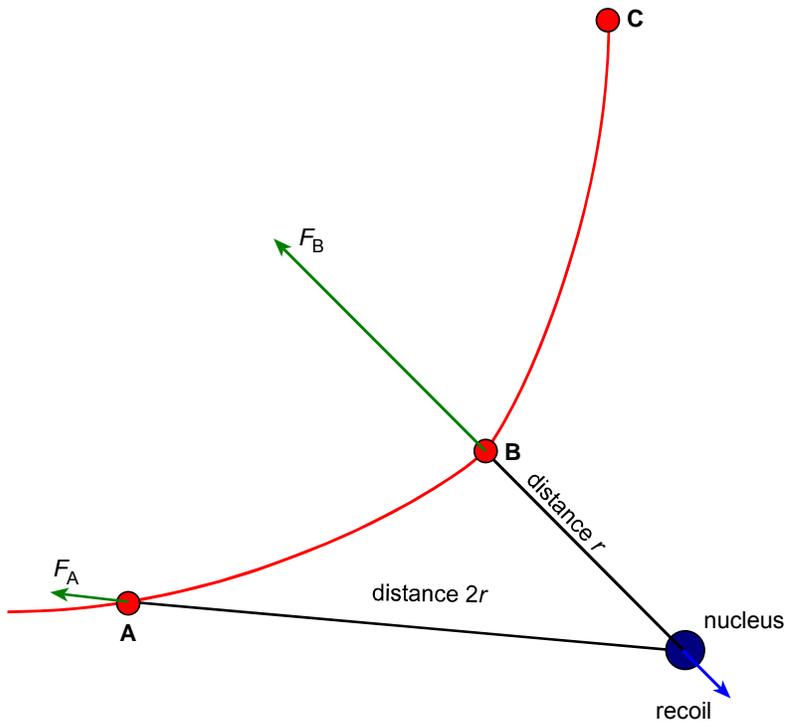
- The force at B is four times as large as the force at A, because the distance is halved and the force varies as  $1/r^2$ .
- The force on the nucleus is equal and opposite to the force on the alpha particle. But because the nucleus is much more massive, it recoils only slightly.

Path of alpha particle scattered by nucleus



5. Particle moves slowest at B, because this is the distance of closest approach, the particle has been decelerating due to repulsive force. After B it accelerates away.
6. Here the particle is furthest from the nucleus, the alpha particle has been accelerated between B and C and is therefore going fastest at C.
7. The alpha particle path is symmetrical about the line from the nucleus to B. So the net change of momentum of the nucleus is along this direction.

Path of alpha particle scattered by nucleus



8. Uphill, because the particle is approaching the nucleus but being pushed away from it.
9. Along contour, because the particle is travelling at right angles to the direction of the force on it.
10. Downhill, because the particle is travelling away from the nucleus and is being pushed away from it.
11. Potential at B larger than potential at A by a factor of 2, because the distance is halved and the potential varies as  $1/r$ .

### External reference

This activity is taken from Advancing Physics chapter 17, 80S

## TAP 522- 4: Rutherford scattering: Energy and closest approach

### Scattering of alpha particles

Rutherford did not have a particle accelerator. Instead he used alpha particles, typically of energy 5 MeV, from radioactive decay. These questions are about how close an alpha particle can get to different nuclei.

An alpha particle has charge  $2e$ , where  $e = 1.60 \times 10^{-19}$  C. A nucleus has charge  $Ze$ , where  $Z$  is the number of protons in the nucleus (and the number of electrons in the atom). The electrical potential energy of the two charges at a distance  $r$  is:

$$\text{electrical potential energy} = \frac{2e \times Ze}{4\pi\epsilon_0 r}$$

where  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1}$ .

The electrical potential energy in electron volts is obtained by dividing by  $1.60 \times 10^{-19} \text{ J eV}^{-1}$

### Calculating the potential energy

1. Show that the units of energy from the expression

$$\text{electrical potential energy} = \frac{2e \times Ze}{4\pi\epsilon_0 r}$$

are joules.

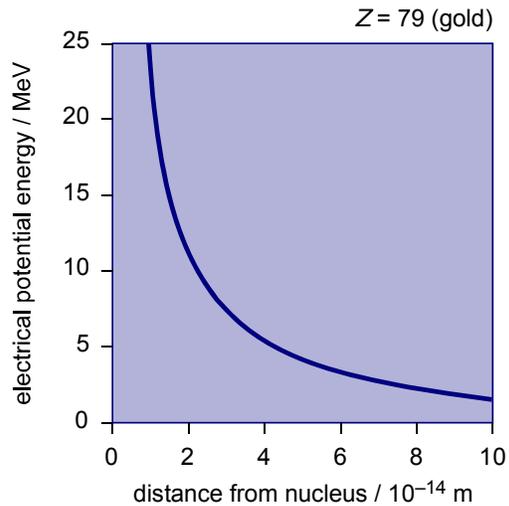
2. Show that the energy in MeV is given by

$$\frac{2Ze}{4\pi\epsilon_0 r} \times 10^{-6}.$$

### Alpha scattering by gold

This graph shows the energy in MeV of an alpha particle at distances  $r$  from a gold nucleus,  $Z = 79$ .

### Approach of alpha particle to nucleus

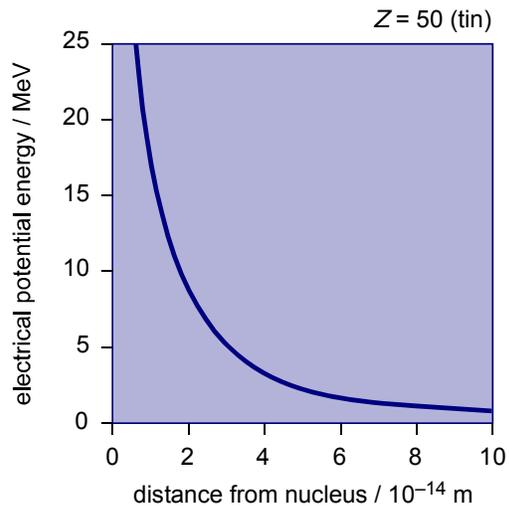


3. Make an arithmetical check to show that at distance  $r = 1.0 \times 10^{-14}$  m the electrical potential energy is between 20 MeV and 25 MeV, as shown by the graph.
  
4. How does the electrical potential energy change if the distance  $r$  is doubled?
  
5. From the graph, at what distance  $r$  will an alpha particle with initial kinetic energy 5 MeV colliding head-on with the nucleus, come to rest momentarily?
  
6. From the graph, at what distance  $r$  will a 5 MeV alpha particle have lost half its initial kinetic energy?
  
7. From the graph, what energy would an alpha particle need to approach as close as  $2.0 \times 10^{-14}$  m in a head-on collision?

## Alpha scattering by tin

The next graph shows, on the same scale as before, the potential energy of an alpha particle near a nucleus of tin,  $Z = 50$ .

### Approach of alpha particle to nucleus

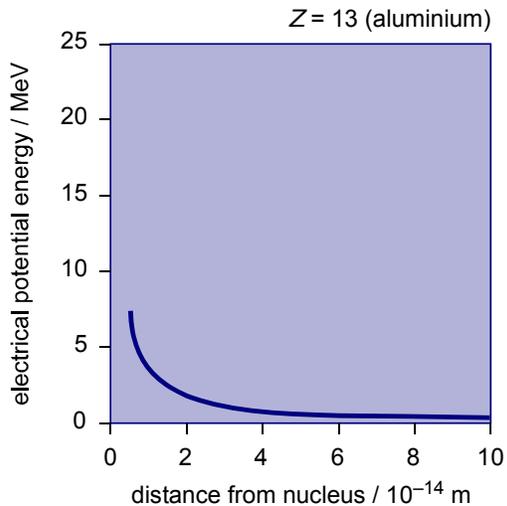


8. Why are the values of the potential energy smaller at the same values of  $r$  in this second graph?
  
  
  
  
  
  
  
  
  
  
9. At  $r = 5.0 \times 10^{-14}$  m the electrical potential energies of an alpha particle are 4.55 MeV for gold,  $Z = 79$  and 2.88 MeV for tin,  $Z = 50$ . Explain the ratio, 1.58, of the two energies.
  
  
  
  
  
  
  
  
  
  
10. Approximately how close can a 5 MeV alpha particle get to a tin nucleus, in a head-on collision?

## Alpha scattering by aluminium

The next graph shows the potential energy of an alpha particle close to an aluminium nucleus,  $Z = 13$ .

### Approach of alpha particle to nucleus



11. From the graph, how close could a 5 MeV alpha particle get to a nucleus of charge  $Z = 13$ , in a head-on collision?
12. The radius of an aluminium nucleus is approximately  $3 \times 10^{-15}$  m. Does a 5 MeV alpha particle get close to the nucleus, compared with the dimensions of the nucleus itself? Could the pattern of scattering be affected?

## Heavy ion colliders

Recently, to investigate very high-energy collisions, accelerators have been used to make head-on collisions between lead nuclei travelling in opposite directions.

13. How much kinetic energy is needed to get two lead nuclei,  $Z = 82$ , within  $1.0 \times 10^{-14}$  m of one another? (Assume that electrical forces are the only forces operating.)

## Hints

1. Treat units like algebraic quantities in the expression for potential energy.
2. Remember that the charge  $e$  coulomb is also the conversion joule per electron volt.
3. Substitute values in the equation for potential energy.
4. Remember  $1/r$ .
5. Read approximately from the graph.
6. Read approximately from the graph.
7. Read approximately from the graph.
8. Look at the equation for electrical potential energy.
9. Try the ratio  $79/50$ .
10. Read approximately from the graph.
11. Read approximately from the graph.
12. Remember that  $10^{-15}$  is  $1/10$  of  $10^{-14}$ .
13. Substitute in the expression for electrical potential energy. Or start with the answer to question 3.

## Practical advice

The questions focus on the distance of closest approach of an alpha particle to a nucleus. The approach is through the shape of the  $1/r$  curve of electric potential energy, and the way the curve varies with radius and charge.

## Alternative approaches

Students could explore the electric potential energy close to a nucleus, using a spreadsheet.

## Social and human context

With hindsight, Rutherford was very clever to have managed without a particle accelerator. But how could he have raised the money to build one without knowing what he would find?

## Answers and worked solutions

1. The units are:

$$\frac{\text{C} \times \text{C}}{\text{C}^2 \text{ J}^{-1} \text{ m}^{-1} \times \text{m}} = \text{J}.$$

2. In the expression

$$\text{electrical potential energy} = \frac{2e \times Ze}{4\pi\epsilon_0 r}$$

dividing by  $e$  gives

$$\frac{2Ze}{4\pi\epsilon_0 r}$$

for the energy in eV. Multiply by  $10^{-6}$  to get the energy in MeV.

3. Substituting values gives

$$E_p = \frac{2 \times 79 \times 1.6 \times 10^{-19} \text{ C}}{4\pi \times 8.85 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1} \times 1.0 \times 10^{-14} \text{ m}} \times 10^{-6} = 22.7 \text{ MeV}.$$

4. Halves, because the potential energy is proportional to  $1/r$ .
5. About  $4.6 \times 10^{-14}$  m, where the graph reaches 5 MeV.
6. Just less than  $10.0 \times 10^{-14}$  m.
7. About 12 MeV.
8. The charge on the nucleus is smaller, so the potential energy is smaller in the same ratio.
9. The ratio of the charges,  $79 / 50 = 1.58$ .

10. About  $3 \times 10^{-14}$  m.
11. About  $0.75 \times 10^{-14}$  m.
12. The alpha particle approaches to about 2.5 times the radius of the nucleus. Attractive forces between nucleons might begin to be important, and modify the scattering.
13. Inserting values:

$$E_p = \frac{82 \times 82 \times 1.6 \times 10^{-19} \text{ C}}{4\pi \times 8.85 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1} \times 1.0 \times 10^{-14} \text{ m}} \times 10^{-6} = 967 \text{ MeV}.$$

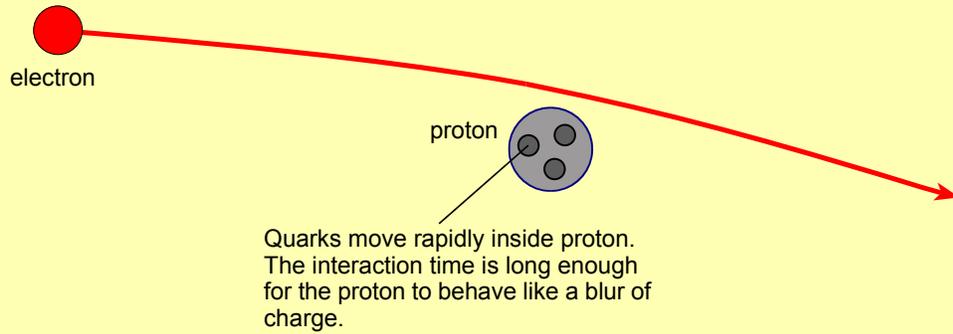
### External reference

This activity is taken from Advancing Physics chapter 17, 70S

## TAP 522-5: Deep inelastic scattering

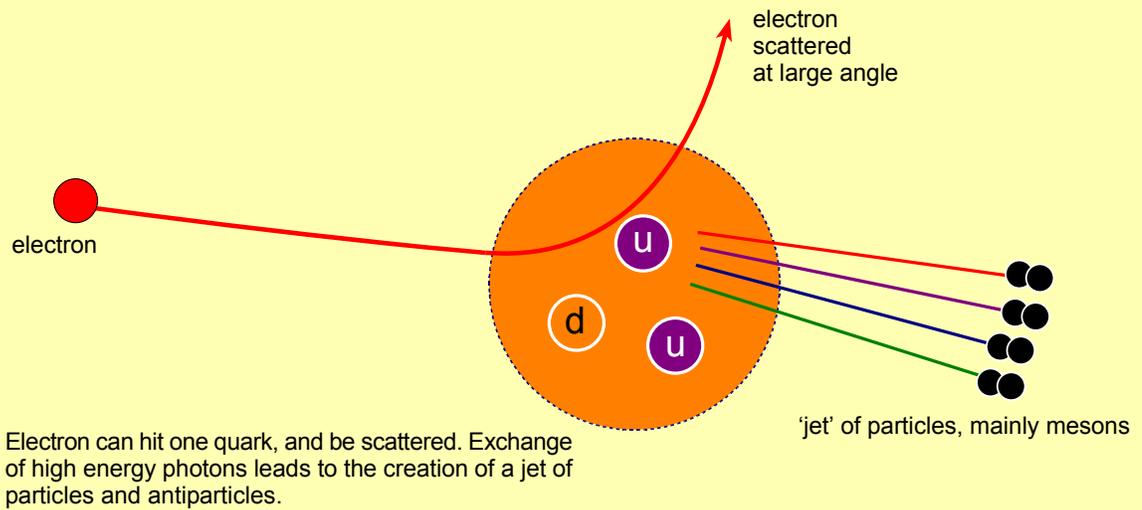
### Deep inelastic scattering

#### Medium energy: elastic scattering

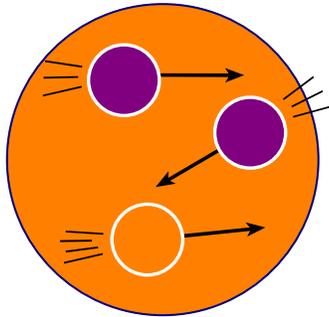


### Deep inelastic scattering

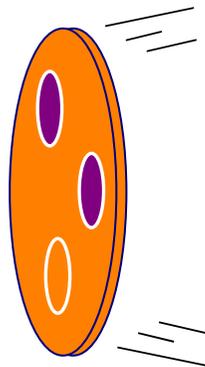
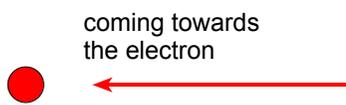
#### High energy: deep inelastic scattering



## Quarks as relativistic stationary pancakes



proton as seen by observer not moving relative to it:  
rapidly moving spherical quarks fill a sphere



proton as seen by electron moving rapidly towards it:  
almost stationary pancake quarks filling a flat disk

**Practical advice**

The diagram could be used as an OHT and discussed in class

**External reference**

This activity is taken from Advancing Physics chapter 17, 1500

## TAP 522-6: Electrons measure the size of nuclei

### Scattering by small particles

Hold a glass plate smeared with a little milk, or dusted with lycopodium powder, in front of a point source of light and you may see rings of light round the source. The photons are diffracted by globules of fat in the milk or by the lycopodium spores.

Similarly to diffraction by a small hole of diameter  $d$ , there is a first minimum intensity at an angle  $\theta$  of order of magnitude given by  $\sin \theta = \lambda / d$ . (For circular objects or apertures a more exact expression is  $\sin \theta = 1.22 \lambda / d$ .)

Angles and wavelengths

1. Show that if you see a first dark ring at  $\theta = 30^\circ$ , the circular objects have diameter approximately twice the wavelength.
2. Use the expression  $\sin \theta = 1.22 \lambda / d$  to find the angle of the first dark ring for particles four wavelengths in diameter.

### Wavelengths for electrons

The de Broglie wavelength  $\lambda$  of an electron of momentum  $p$  is given by  $\lambda = h / p$ , where  $h$  is the Planck constant,  $6.6 \times 10^{-34} \text{ J Hz}^{-1}$ . Since the rest energy of an electron is 0.5 MeV, at energies of hundreds of MeV, the rest energy can be ignored as part of the total energy  $E$ . In this case the momentum  $p$  is given to a good approximation by  $p = E / c$ .

3. Calculate the energy in joules of an electron with energy 100 MeV  
(take  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ).
4. Use the value of the energy from question 3 to calculate the momentum of the electron.
5. Use the value of the momentum from question 4 to calculate the de Broglie wavelength of 100 MeV electrons.

6. The radius of a single proton or neutron is of the order  $1.2 \times 10^{-15}$  m. What approximately is the ratio of the wavelength of the electrons to the diameter of a proton or neutron?
  
7. Using the relations  $p = E / c$  and  $\lambda = h / p$  show that the de Broglie wavelength is inversely proportional to the energy  $E$ .
  
8. Use the result of question 7 and the answer to question 5 to show that the de Broglie wavelength for 400 MeV electrons is about  $3.0 \times 10^{-15}$  m.

### **Electron scattering by nuclei**

You have seen that electrons of a few hundred MeV have de Broglie wavelengths comparable to the diameter of a nucleus. Suppose that in an experiment a beam of 400 MeV electrons is scattered by carbon-12 nuclei. The angle  $\theta$  at which the scattering is first a minimum is  $42^\circ$ , for which  $\sin \theta = 0.67$ .

9. Calculate the ratio  $\lambda / d$  of the de Broglie wavelength to the diameter of a carbon-12 nucleus.
  
  
  
  
  
  
  
  
  
  
10. Use the de Broglie wavelength of 400 MeV electrons from question 8 to show that the radius of a carbon-12 nucleus is about  $2.7 \times 10^{-15}$  m.
  
  
  
  
  
  
  
  
  
  
11. You might expect the volume occupied by the 12 nucleons of carbon-12 to be 12 times the volume occupied by one nucleon. The radius of a nucleon is about  $1.2 \times 10^{-15}$  m. Show that the ratio of the volumes is about 12 (expect some rounding error in these figures).

12. A uranium-238 nucleus has a radius of about  $7.4 \times 10^{-15}$  m. What roughly would be a good energy of electrons to use to determine its radius by scattering?

### Hints

1.  $\sin 30^\circ = \frac{1}{2}$ .
2. Substitute in the expression for  $\sin \theta$ .
3. The conversion factor is equal to the magnitude of the charge on the electron.
4. Use  $p = E/c$ .
5. Use  $\lambda = h/p$ .
6. Remember that the diameter = 2 x radius.
7. Substitute  $p = E/c$  for  $p$ .
8. Scale down the wavelength in proportion to the increase in energy.
9. Obtain the ratio  $\lambda/d$  from the value of  $\sin \theta$ .
10. Remember the radius is half the diameter.
11. The ratio of the volumes is the cube of the ratio of the radii.
12. Choose a reasonable angle, say  $30^\circ$ .

## Practical advice

These questions lead students through a numerical example of the measurement of nuclear dimensions by electron scattering, using the diffraction of the electrons to obtain the scale. Many students will require help with the powers of ten involved, and the conversion between electron volts and energy in joules.

The questions are intended primarily as 'learning questions' to be gone through slowly. Some students may profit from tackling them alone, but most will need to be taken through them, and have the general message pointed out for them.

## Social and human context

To measure these tiny distances needed a linear accelerator a few kilometres in length. Large energies to measure small dimensions are expensive.

## Answers and worked solutions

1.  $\sin 30^\circ = \frac{1}{2} = \lambda/d$  approximately.

2.

$$\begin{aligned}\sin \theta &= 1.22 \frac{\lambda}{d} \\ &= \frac{1.22}{4} \\ &= 0.305.\end{aligned}$$

The angle whose sin is 0.305 is  $17.8^\circ$ .

3.

$$\begin{aligned}\text{Energy} &= 100 \text{ MeV} \\ &= 10^8 \text{ eV} \\ &= 10^8 \text{ eV} \times (1.6 \times 10^{-19} \text{ J eV}^{-1}) \\ &= 1.6 \times 10^{-11} \text{ J}.\end{aligned}$$

4.

$$\begin{aligned}p &= \frac{E}{c} \\ &= \frac{1.6 \times 10^{-11} \text{ J}}{3.0 \times 10^8 \text{ m s}^{-1}} \\ &= 0.53 \times 10^{-19} \text{ kg m s}^{-1}.\end{aligned}$$

5.

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{6.6 \times 10^{-34} \text{ J s}}{0.53 \times 10^{-19} \text{ J m}^{-1} \text{ s}} \\ &= 12 \times 10^{-15} \text{ m}.\end{aligned}$$

6. Ratio of wavelength to radius =  $12 \times 10^{-15} \text{ m} / 1.2 \times 10^{-15} \text{ m} = 10$ ;  
ratio of wavelength to diameter = 5.

7. Substitute

$$p = \frac{E}{c}$$

in

$$\lambda = \frac{h}{p}$$

gives

$$\lambda = \frac{hc}{E}.$$

8. From

$$\lambda = \frac{hc}{E}$$

$\lambda$  is inversely proportional to  $E$ . Since  $\lambda$  for 100 MeV is  $12 \times 10^{-15} \text{ m}$  then  $\lambda$  for 400 MeV is  $3 \times 10^{-15} \text{ m}$ .

9. Since  $\sin \theta = 0.67$ , then  $0.67 = 1.22 \lambda / d$ , and  $\lambda / d = 0.55$ .

10. Since  $\lambda$  for 400 MeV is  $3 \times 10^{-15} \text{ m}$ , and  $\lambda / d = 0.55$ , then  $d = 5.5 \times 10^{-15} \text{ m}$  and  $r = 2.7 \times 10^{-15} \text{ m}$ .

11.  $(2.7/1.2)^3 = 11.4$ . It would be closer to 12 but for rounding errors.

12. Diameter of the uranium-238 nucleus =  $15 \times 10^{-15} \text{ m}$ . Choosing  $\lambda = 2d$  gives  $\lambda = 30 \times 10^{-15} \text{ m}$ , about five times the wavelength of 400 MeV electrons ( $3 \times 10^{-15} \text{ m}$ ). So scale down energy by a factor 5, to say 80 MeV.

## **External reference**

This activity is taken from Advancing Physics chapter 17, 90S