

TAP 535- 4: Creation and annihilation

Using $E_{\text{rest}} = m c^2$

These questions give practice in using $E_{\text{rest}} = m c^2$ to calculate photon energies and masses of particles created or annihilated.

Energy and mass

The energy of a particle at rest is all due to its mass. This energy is called the rest energy. The rest energy in joules of a particle with mass m measured in kilograms is given by

$E_{\text{rest}} = m c^2$, where c is the speed of light in metres per second. An energy in joules can be converted to electron volts by dividing by $1.60 \times 10^{-19} \text{ J eV}^{-1}$.

The mass of an electron or positron is $9.11 \times 10^{-31} \text{ kg}$. The speed of light is $3.00 \times 10^8 \text{ m s}^{-1}$.

1. Show that the rest energy of an electron is $8.2 \times 10^{-14} \text{ J}$.
2. Use the answer to question 1 to show that the rest energy of an electron is 0.51 MeV.
3. Write down the rest energy of a positron (antielectron).
4. An electron and a positron which meet annihilate one another. By how much does the rest energy decrease in total? Express the answer in MeV.
5. The annihilation of an electron and a positron at rest produces a pair of identical gamma ray photons travelling in opposite directions. Write down in MeV the energy you expect each photon to have
6. A single photon passing near a nucleus can create an electron–positron pair. Their rest energy comes from the energy of the photon. Write down the smallest photon energy that can produce one such pair.

7. Cosmic rays can send high-energy photons through the atmosphere. What approximately is the maximum number of electron–positron pairs that a 10 GeV photon can create?

8. The isotope $^{24}_{12}\text{Mg}$ is stable. The light isotope $^{22}_{12}\text{Mg}$ emits positrons and gamma rays including a photon of energy 1.28 MeV. How can decays of this nucleus result in both annihilation and creation of electron–positron pairs?

9. A photon can create particle–antiparticle pairs of greater mass than electrons and positrons. Approximately what energy must a photon have to create a proton–antiproton pair? (The mass of a proton is 2000 times the mass of an electron).

10. Why do the photons from the annihilation of an electron–positron pair not themselves go on to create new electron–positron pairs?

Practical advice

These are intended as simple practice calculations on energy changes in particle–antiparticle creation and annihilation.

Alternative approaches

You might start in the context of PET scans, which use positron annihilation.

Social and human context

The existence of the creation and annihilation of matter makes for a profound shift in the way the world has to be imagined. Its permanence is undermined.

Answers and worked solutions

1.

$$E_{\text{rest}} = mc^2$$
$$E_{\text{rest}} = 9.11 \times 10^{-31} \text{ kg} \times (3.00 \times 10^8 \text{ ms}^{-1})^2$$
$$= 8.2 \times 10^{-14} \text{ J}$$

2. Energy in eV is energy in joules divided by $1.60 \times 10^{-19} \text{ J eV}^{-1}$:

$$\frac{8.2 \times 10^{-14} \text{ J}}{1.60 \times 10^{-19} \text{ J eV}^{-1}} = 5.1 \times 10^5 \text{ eV} = 0.51 \text{ MeV}$$

3. 0.51 MeV since electrons and positrons have identical mass.
4. 1.02 MeV. Two particles, each of rest energy 0.51 MeV, are annihilated.
5. 0.51 MeV each. The two photons share the total 1.02 MeV.
6. 1.02 MeV. The photon has to create the rest energy of two particles, each 0.51 MeV.
7. Approximately 10^4 pairs. Each pair requires approximately 1 MeV = 1×10^6 eV and 10 GeV = 10×10^9 eV is available.
8. The 1.28 MeV photon has enough energy to create an electron–positron pair. The positron from the decay can annihilate with any nearby electron.
9. Approximately 2 GeV. The rest energy of a proton–antiproton pair is 2000 times greater than the roughly 1 MeV rest energy of an electron–positron pair.
10. Their 0.51 MeV energy is too small (minimum 1.02 MeV).

External references

This activity is taken from Advancing Physics chapter 17, 30S