TAP 705- 2: Calculating the age of the Universe

Hubble's constant

The Hubble constant can be used to estimate the age of the Universe. Over time, estimates of this constant have changed and so estimates of the age of the Universe have changed. This question asks you to work out the changing estimates of the age of the Universe from the Hubble constant.

What does the constant mean?

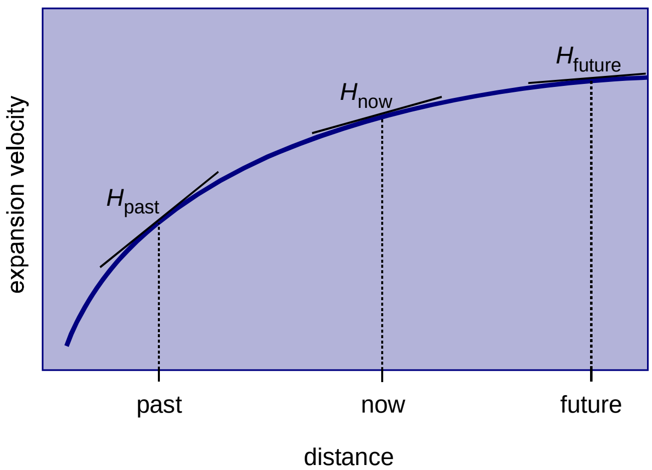
Hubble's realised that the light from distant galaxies is red-shifted and that the more distant they are, the greater the red shift. Today, this is interpreted to mean that the Universe is expanding: that is, the space between clusters of galaxies is expanding and therefore the distance between galaxies is increasing. Hubble's interpretation was slightly different: he took the results to mean that galaxies were receding from each other through space. Following Hubble's explanation, we can write:

$$v = Hd$$

where v is the recession speed of a distant galaxy, d its distance from us and H is the Hubble constant. Now, if you imagine the Universe running backwards, how long would it take a distant galaxy to reach you? Answering this question tells you how long ago it is since all the galaxies were together in the same place, i.e. how long ago the Big Bang occurred. The time taken for a galaxy travelling at speed v to travel a distance d is:

$$\frac{d}{v} = \frac{1}{H}.$$

Therefore, the value of 1 / H gives us an estimate of the age of the Universe. This is only as an estimate because as the galaxies in the time-reversed Universe fall towards one another, they would be expected to speed up.

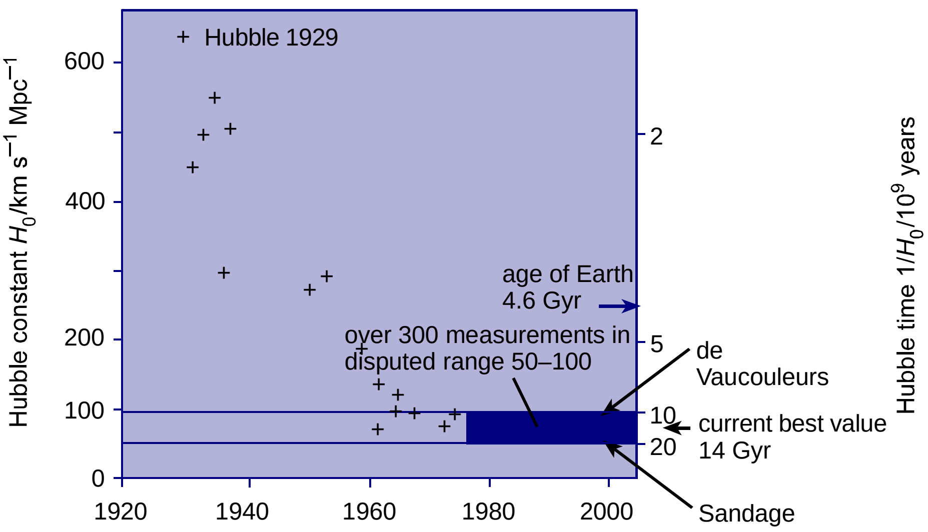


A more precise description of 1 / H is the 'expansion timescale', or 'Hubble time' rather than the 'age of the Universe'.

A graph of measured values of Hubble's constant

The following graph, taken from *Physics World*, July 1999, shows how measured values of the Hubble constant have changed since the 1920s, when Hubble made his original measurement.

How estimates of the Hubble time have changed



The units of H on the graph are km s^{-1} Mpc^{-1} (km per second per million parsecs). This is a useful unit for astronomers as it tells them how fast, in km s^{-1}, a galaxy is receding if its distance is a certain number of parsecs from us. The two different units are useful to astronomers for measuring the two different quantities, speed and distance, but if you want to find the age of the Universe from 1 / H, then you must have H in units of (time)$^{-1}$. The first thing you have to do is change the units of H into s^{-1}.

1. What is 1 Mpc in km?

2. By writing 1 km s^{-1} Mpc^{-1} in terms of km s^{-1} km^{-1}, write down a conversion factor that changes km s^{-1}Mpc^{-1} into s^{-1}.

3. The earliest point on the graph gives a Hubble constant value of approximately 600 km s^{-1}Mpc^{-1}. What is this in s^{-1}?

4. For this early value of the Hubble constant, calculate an estimate for the age of the Universe. What are the units of your answer? Convert the answer to years.

5. Read from the graph the upper and lower limits of the recent disputed values of the Hubble constant. Calculate the two different ‘ages of the Universe’ that they imply.

Hints

1. 1 parsec is 3.09×10^{16} m

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2. Use your answer to question 1 to write 1 Mpc in km. The reciprocal of this gives the conversion factor – note that the km cancel out when you rewrite the unit.
3. Use your conversion factor from question 2.
4. Simply find the reciprocal of your answer to question 3 and, noting that the answer must therefore be in seconds, you can easily convert to years by calculating how many seconds there are in a year.
5. This is simply lots of calculations like question 4. Putting a formula in a spreadsheet means that you can do all the calculations at one go.

Practical advice

This exercise is intended to give practice in converting Hubble's constant to an age of the Universe. It is important for students to understand the connection between this constant and the age of the Universe, and this point will need careful explanation for some students. Important points can also be made about units: from Hubble's equation, H should have dimension $(\text{time})^{-1}$, yet astronomers use an odd unit (which, of course, does have the correct dimensions) for convenience. It is not sensible to use the same length unit in the speed and in the distance to distant galaxies.

First, some agreement is being reached on the value of H and therefore of the age of the Universe – a remarkable achievement. Second, large numbers of people have kept themselves busy measuring this constant. Third, the range from 50 to 100 km s^{-1} Mpc^{-1} was the subject of a bitter dispute over at least a decade, only recently resolved. In fact values as low as 42 have been claimed.

Finally, make the point that H is not a constant at all! As the Universe expands, its rate of expansion will change (it will fall if the cosmological constant is zero; it may increase if the cosmological constant is non-zero). In the absence of the cosmological constant, the values of the age of the Universe calculated here give only an upper limit.

Social and human context

Should we care about the age of the Universe? Should society spend money to measure the age of the Universe? Why are such questions important to some? Why has the origin of the Universe fascinated humans for so long?

Answers and worked solutions

1. 1 Mpc = 3.09×10^{19} km
2. 1 Mpc = 3.09×10^{19} km. Therefore:

$$
\begin{aligned}
1\,\text{km s}^{-1}\,\text{Mpc}^{-1} &= 1\,\text{km s}^{-1}(3.09 \times 10^{19}\ \text{km})^{-1} \\
&= 1\,\text{km s}^{-1}\,\text{km}^{-1} \times (3.09 \times 10^{19})^{-1} \\
&= 3.24 \times 10^{-20}\ \text{km s}^{-1}\,\text{km}^{-1} \\
&= 3.24 \times 10^{-20}\ \text{s}^{-1}.
\end{aligned}
$$

3.

$$600\ \text{km s}^{-1}\ \text{Mpc}^{-1} = 600 \times (3.24 \times 10^{-20})\ \text{s}^{-1}$$
$$= 1.94 \times 10^{-17}\ \text{s}^{-1}$$

4. Age of the Universe = 1/H:

$$1/H = (1.94 \times 10^{-17}\ \text{s}^{-1})^{-1}$$
$$= 5.14 \times 10^{16}\ \text{s}$$
$$= \frac{5.14 \times 10^{16}\ \text{s}}{60 \times 60 \times 24 \times 365\ \text{s year}^{-1}}$$
$$= 1.63\ \text{billion years.}$$

5. The values in dispute lie between 100 km s^{-1} Mpc^{-1} and 50 km s^{-1} Mpc^{-1}. The ages implied lie between approximately 10 billion years and 20 billion years.

External reference

This activity is taken from Advancing Physics, chapter 12, 90D