# TAP 702- 6: Binary stars

Orbiting binary stars:

A type of variable star.

This type of variable star consists of two stars orbiting around each other. When the dimmer star is in front of the brighter one, the observed intensity is at a minimum, and when the two stars are side by side, the observed intensity is at a maximum.

Relative velocities in an orbiting binary star system.

The two stars in this question are similar in mass, but star B is cooler and dimmer than star A. They orbit about their common centre of gravity X, as shown in the diagram.
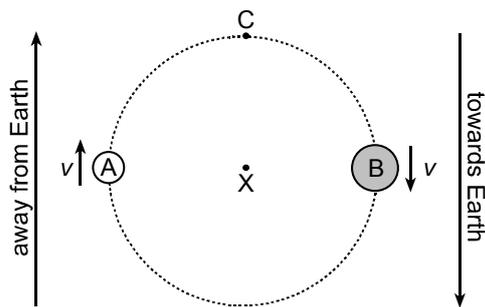
In one such pair of stars, the time for one revolution is 73 days.

The stars are a distance of 1×10^{11} m apart.

1 day = 8.64×10^4 s

1.   What are the velocities of stars A and B, relative to the point X?

2.   What is the velocity of star A, relative to star B?

The point X is not still with respect to Earth. It is moving at a velocity of 64 km s^{-1} in the direction away from Earth.

3.   What are the velocities of stars A and B, relative to Earth?

4.   What is the velocity of either star, relative to Earth, when it is at the position C?

The Doppler shift in wavelength for a star moving at a velocity v relative to the observer is

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

if the speed v is small compared with the speed of light, c. The speed of light, c,

is 3.0×10^8 m s^{-1}.

5.　　Find the wavelength shift $\Delta\lambda$ that an astronomer (on Earth) would observe in a spectral line of wavelength 589.0 nm from star A in the position shown in the diagram. Is this a red shift or a blue shift?

6.　　Find the wavelength shift $\Delta\lambda$ that an astronomer (on Earth) would observe in a spectral line of wavelength 589.0 nm from star B in the position shown in the diagram. Is this a red shift or a blue shift?

7.　　Find the wavelength shift $\Delta\lambda$ that an astronomer (on Earth) would observe in a spectral line of wavelength 589.0 nm from either star when it is in the position C on the diagram. Is this a red shift or a blue shift?

8.　　In many binary stars, the two stars are not perfectly lined up when seen from Earth. This means that there will not be any dimming or brightening of the light, because the dimmer star will not block out the light from the brighter one. How might an astronomer tell, from the spectrum, that there are in fact two stars moving about their common centre of mass as described in this question?

Hints

1.　　You can quickly find out how far each star goes in 73 days. Don't forget that velocity is measured in metres per second (m s^{-1}), and it has a direction.

2.　　Imagine you are on Star B. How fast, and in which direction, does star A seem to move?

3.　　If the binary star is moving, it means that the point X is moving at that velocity. You can easily combine the answers to part 1 with 64 km s^{-1} (taking care of units!) to get the answer here.

4.　　You need to combine 64 km s^{-1} with the component, in the direction towards / away from Earth, of the velocity of a star when at C.

5.　　This question, and the following two, follow from the results of question 3 and 4. Red shifts are due to stars moving away from Earth, blue shifts are due to stars approaching.

6.　　Is it approaching, or retreating? Is the wavelength being stretched, or squashed?

7. If you've done question 6, this should be fairly obvious.

8. Although the intensity will not change with time, the spectrum will – provided that the stars are not moving perpendicular to the line of view.

Practical advice

Fairly straightforward applications; difficulties arise in visualising the movement and the magnitudes of distances and velocities involved. Note that some binary stars, e.g. Capella (Aurigae) orbit in a plane perpendicular to our line of sight, and so do not show this phenomenon. The formula used for the wavelength shift

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

is justified due to the low speeds involved. Note that there is a relativistic transverse Doppler shift (a time dilation effect) that affects the answer to question 7. However, again due to the small speeds involved, this effect is negligible.

Algol (β Persei) is the classic eclipsing system; with a period of about 3 days. Details are given in most astronomy books, e.g. Norton's 2000, or virtually any encyclopaedia.

Or see:

http://www.solstation.com/stars2/algol3.htm

http://en.wikipedia.org/wiki/Beta_Persei

http://domeofthesky.com/clicks/algol.html

Alternative approaches

A battery-operated buzzer in a net bag, tied to a strong cord and whirled around the head, shows this effect quite nicely. Note that the Doppler effect for sound is dependent on the motion of the medium so that it is not strictly the same as the Doppler effect considered here. Safety note: do not smack anyone around the head with the whirling buzzer!

TAP 702-1: The Doppler effect

Social and human context

Historical perspective: the Doppler measurements are typical of the sort of discoveries made with the Doppler effect, once the resolution of the spectrometers was up to fractions of a nanometre.

Answers and worked solutions

1. The speed of stars in orbit is equal to $\pi D / T$:

$$v = \frac{\pi D}{T} = \frac{\pi \times (1 \times 10^{11}\,\text{m})}{73\,\text{days} \times 8.6 \times 10^4\,\text{s day}^{-1}} = 5.0 \times 10^4\,\text{m s}^{-1}$$

Therefore the velocity of A relative to X is $+ 5.0 \times 10^4$ m s^{-1} and the velocity of B relative to X is $- 5.0 \times 10^4$ m s^{-1} where the positive direction is the direction away from the Earth.

2. The velocity of A relative to B is $(+ 5.0 \times 10^4$ m s$^{-1}) - (- 5.0 \times 10^4$ m s$^{-1})$ which is equal to $+ 1.0 \times 10^5$ m s^{-1} with directions defined as in question 1.

3. Adding the velocities in question 1 to 6.4×10^4 m s^{-1} gives

$$\text{Velocity of A relative to Earth} = + 5.0 \times 10^4 \text{ m s}^{-1} + 6.4 \times 10^4 \text{ m s}^{-1}$$
$$= 1.1 \times 10^5 \text{ m s}^{-1}.$$

$$\text{Velocity of B relative to Earth} = - 5.0 \times 10^4 \text{ m s}^{-1} + 6.4 \times 10^4 \text{ m s}^{-1}$$
$$= 1.4 \times 10^4 \text{ m s}^{-1}$$

with directions defined as in question 1.

4. Here the component of the velocity of each star, relative to X, in the direction defined in question 1, is zero, so each star has a velocity relative to Earth which is the same as that of X, i.e. 6.4×10^4 m s^{-1}.

5.

$$\Delta\lambda = \frac{\lambda v}{c}$$
$$= \frac{(589.0 \times 10^{-9} \text{ m}) \times (1.1 \times 10^5 \text{ m s}^{-1})}{3.0 \times 10^8 \text{ m s}^{-1}}$$
$$= 2.2 \times 10^{-10} \text{ m}$$
$$= 0.22 \text{ nm}$$

This is a red shift, because the star is moving away from the Earth, and so the wavelength is 'stretching'.

6.

$$\Delta\lambda = \frac{\lambda v}{c}$$
$$= \frac{(589.0 \times 10^{-9} \text{ m}) \times (1.4 \times 10^4 \text{ m s}^{-1})}{3.0 \times 10^8 \text{ m s}^{-1}}$$
$$= 2.8 \times 10^{-11} \text{ m}$$
$$= 0.028 \text{ nm}$$

This is a red shift for the same reason as question 5.

7.

$$\Delta\lambda = \frac{\lambda v}{c}$$
$$= \frac{(589.0 \times 10^{-9} \text{ m}) \times (6.4 \times 10^4 \text{ m s}^{-1})}{3.0 \times 10^8 \text{ m s}^{-1}}$$
$$= 1.3 \times 10^{-10} \text{ m}$$
$$= 0.13 \text{ nm}$$

This is also a red shift for the same reason as question 5.

8.   When the stars are at C and the point diametrically opposite to C respectively, then both will have their 589.0 nm spectral line red shifted by 0.13 nm, as both will have the same velocity relative to the Earth (question 4). When they move towards A and B, however, their velocities relative to Earth increase and decrease respectively, causing the spectral line to be red shifted by different amounts, so it will split into two distinct spectral lines. These will merge into one as the stars line up in the direction of Earth once more.

Should the two stars be moving in a circle whose plane is perfectly perpendicular to the Earth, however, like the stars in the Capella binary system, then there will be no splitting of the line, as the velocity relative to Earth in the perpendicular direction will not change during the orbit.

External References

This activity is taken from Advancing Physics, chapter 12, 60S